Electrical Machines II

Dynamic Behavior, Converter Supply and Control

based on a lecture of

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at

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Preface

These notes represent the state of the lecture “Electrical Machines II” at Aachen University for the summer term effective from 2003. Some extensions of the subject matter, which are beyond the scope of the lecture, are included for self-studies at suitable location.
In the lecture the dynamic behavior of the DC-, induction- and synchronous- machine are discussed. Also voltage and frequency variable excitation with power converters and control methods needed for power generation and electrical drives are treated. In addition to the analytical solution of the differential equations under simplifying assumptions, numerical solution by means of computers will be demonstrated. Also these simulations take the power converter and control system into account, which is shown on practical applications.
The areas basics, mode of operation, structure and steady state operating behavior are topics of the lecture “Electrical Machines I”, which is subject to the lecture held in winter terms.
Focus is put on provision of clear understanding of the physical context. Despite plain description required accuracy is not reduced.
This script is supposed to provide an all-embracing knowledge as a basis for both the continuation of their studies and later in practice to deal with electrical machines in detail.
The lecture “electrical machines II” was especially elaborated for students in main course of the major “Electro-Technique and Electronics”. The knowledge of contents of the lecture “Electrical Machines I” is presumed.

Please note: This script represents a translation of the lecture notes composed in German. Most subscriptions to appear in equations are not subject to translation for conformity purposes.

Aachen, April 2002 Gerhard Henneberger

Revision 01: Busch, Schulte, Löwenstein, Dezember. 02
Revision 02: Schulte, July 2003

Content

1 Direct and quadrature axis theory

1.1 Introduction

Up to now always steady-state operation was presumed, i.e. operation on power system with constant voltage (DC-, AC or three-phase system), constant speed and also without electrical switching operation and mechanical load variation. Such mode of operation nearly never occurs in real world applications, neither at drive engineering nor at central electricity supply.

The electrical machines are the main parts in drive engineering, which are the controlled system in the actuating system. Final controlling element - today mostly a static converter - and analog or digital control complete the drive system with speed control. DC motors are of minor importance nowadays, since recent advances in power semiconductor and microprocessor technology, which increased the relevance of induction- and EC-motors for electrical drives. High dynamic responses at acceleration or braking and a short setting time in case of mechanical load changes are demanded.

In central power supply the fundamental basis is provided by synchronous generators. They are used in thermal power stations in conjunction with steam turbines in form of a high speed cylindrical-rotor generator and in hydro-electric power plants as low speed salient-pole generators. The voltage is preferably sinusoidal and the frequency should be constant. The speed of the driving machine and the terminal voltage must be controlled in a kind of manner, that sudden load change provoked by short circuit fault or cut-off do not result in large changes of the frequencies and voltages. Also operation in parallel with other generators must be possible without the occurrence of oscillations.

The knowledge of the response characteristic of the “controlled system electrical machine” is therefore an important precondition for the design and the prediction of the operational performance of an electrical machine.

While in dynamical operation the transfer function of the electrical and mechanical quantities can always be described by a set of differential equations. The number of these equations depends on the number of energy storage mechanism, i.e. a voltage equation for each coil and for the rotor mass an equation of motion.

To limit the complexity of the calculation for three-phase machine with non constant mutual inductances, some restrictive conditions must be made regarding electrical and magnetic symmetry and fundamental wave. Additionally two kinds of transformation are required. The three-phase winding will be transformed to a two-phase system, which magnetically decouples the rotor and stator windings and also reduces the number of equations per windings from three to two. The other transforms from a stationary to an arbitrary rotating coordinate system. Performing this transformation will result in constant mutual inductances and the feasibility to take magnetic asymmetry of some machine parts into account for instance the different reactances $X_d \neq X_q$ of salient pole machines.

The transformations have to be power invariant and so that resistances and inductivities stay unchanged. The torque differential equation will be derived from the power balance and the torque equilibrium.

The description of polyphase machines requires at least five differential equations, i.e. two for the rotor, two for the stator and one for the rotating masses. Also the dynamic equations for the DC machine arise from the quadrature-axis theory. In that case only three equations are needed for field, armature and mass.

The derivation of a dynamic equation system of the polyphase machine will be accomplished in common by the consequential use of the quadrature-axis theory. The often used representative space vector method results in identical equations but loses clearness if invariance of the power is taken into account. The correlation between quadrature-axis theory and space vector diagram will be pointed out later.

1.2 General rotating field machine

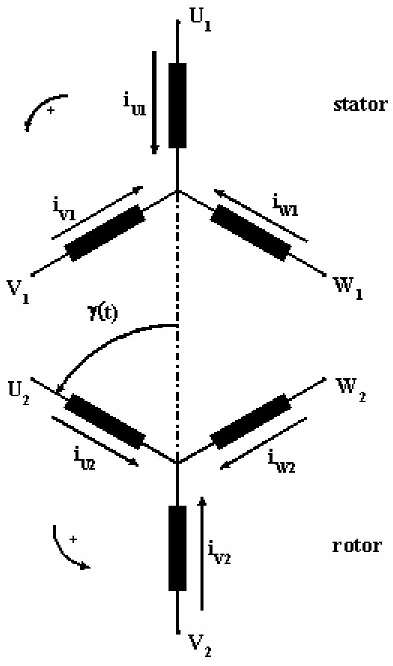


Fig. 1: rotating field

A generalized rotating field machine consists of 3 phase rotor and stator winding (Fig.1). Because the rotor revolves with $g(t)$, the inductances are depending on the rotor position.

The complete set of equations will be presented in matrix form.

voltages:

$$[U_s] = [\underline{u}_{u1}, \underline{u}_{v1}, \underline{u}_{w1}]^T \qquad [U_R] = [\underline{u}_{u2}, \underline{u}_{v2}, \underline{u}_{w2}]^T \qquad (1.1)$$

currents:

$$[I_S] = [\underline{i}_{u1}, \underline{i}_{v1}, \underline{i}_{w1}]^T \qquad [I_R] = [\underline{i}_{u2}, \underline{i}_{v2}, \underline{i}_{w2}]^T \qquad (1.2)$$

flux linkages:

$$[\Psi_S] = [\underline{\Psi}_{u1}, \underline{\Psi}_{v1}, \underline{\Psi}_{w1}]^T \qquad [\Psi_R] = [\underline{\Psi}_{u2}, \underline{\Psi}_{v2}, \underline{\Psi}_{w2}]^T \qquad (1.3)$$

resistances:

$$[R_S] = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_1 & 0 \\ 0 & 0 & R_1 \end{bmatrix} \qquad [R_R] = \begin{bmatrix} R_2 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_2 \end{bmatrix} \qquad (1.4)$$

voltage equations:

$$[U_S] = [R_S] \cdot [I_S] + \frac{d}{dt}[\Psi_S] \qquad [U_R] = [R_R] \cdot [I_R] + \frac{d}{dt}[\Psi_R] \qquad (1.5)$$

flux linkage equations:

$$[\Psi_S] = [L_S] \cdot [I_S] + [M_S] \cdot [I_R] \qquad [\Psi_R] = [L_R] \cdot [I_R] + [M_R] \cdot [I_S] \qquad (1.6)$$

inductances:

$$[L_S]=\begin{bmatrix} L_{1hw}+L_{1s} & L_{1hw}\cdot\cos\left(\frac{2p}{3}\right) & L_{1hw}\cdot\cos\left(-\frac{2p}{3}\right) \\ L_{1hw}\cdot\cos\left(-\frac{2p}{3}\right) & L_{1hw}+L_{1s} & L_{1hw}\cdot\cos\left(\frac{2p}{3}\right) \\ L_{1hw}\cdot\cos\left(\frac{2p}{3}\right) & L_{1hw}\cdot\cos\left(-\frac{2p}{3}\right) & L_{1hw}+L_{1s} \end{bmatrix} \qquad (1.7)$$

$$[L_R]=\begin{bmatrix} L_{2hw}+L_{2s} & L_{2hw}\cdot\cos\left(\frac{2p}{3}\right) & L_{2hw}\cdot\cos\left(-\frac{2p}{3}\right) \\ L_{2hw}\cdot\cos\left(-\frac{2p}{3}\right) & L_{2hw}+L_{2s} & L_{2hw}\cdot\cos\left(\frac{2p}{3}\right) \\ L_{2hw}\cdot\cos\left(\frac{2p}{3}\right) & L_{2hw}\cdot\cos\left(-\frac{2p}{3}\right) & L_{2hw}+L_{2s} \end{bmatrix} \qquad (1.8)$$

mutual inductances:

$$[M_S]=\begin{bmatrix} M_W\cdot\cos(g) & M_W\cdot\cos\left(g+\frac{2p}{3}\right) & M_W\cdot\cos\left(g-\frac{2p}{3}\right) \\ M_W\cdot\cos\left(g-\frac{2p}{3}\right) & M_W\cdot\cos(g) & M_W\cdot\cos\left(g+\frac{2p}{3}\right) \\ M_W\cdot\cos\left(g+\frac{2p}{3}\right) & M_W\cdot\cos\left(g-\frac{2p}{3}\right) & M_W\cdot\cos(g) \end{bmatrix} \qquad (1.9)$$

$$[M_R]=[M_S]^T \qquad (1.10)$$

L_W and M_W are alternating inductances. The number of turns of stator and rotor windings differ.

In the form presented above the dynamic set of equations is physically complex and it is quite complicated to deal with it without using computers.

1.3 Requirements and approach

The following requirements are be made for the quadrature axis theory for simplification purposes:

1. Rotor and stator winding only excite spatial sinusoidal current linkages. That means, that only the fundamental wave of the current linkages is taken into account and winding factors of all harmonics are supposed to be zero.
2. There is no saturation, i.e. the magnetic conductance is independent of the current linkage. The magnetic voltage drop is negligible.
3. The machine is fully symmetric, i.e. constant air gap around the whole circumference and the influence of the slotting is negligible. Within the rotor or stator the self-and mutual inductances are independent from the rotor position This precondition may be partly ignored. If the stator respectively the rotor have two magnetic or electric preferred perpendicular axes, the quadrature axis theory can still be applied through the choice of an asymmetric part fixed coordinate system.
4. The neutral point is not connected. Therewith the number of voltage equations in rotor and stator will be reduced from three to two.

$$i_u + i_v + i_w = 0 \Rightarrow i_w = -i_u - i_v \tag{1.11}$$

Also this precondition can be bypassed by separation and extra handling of the zero phase-sequence system.

The further approach is as follows:

1. Power invariant transformation of both three-phase systems (rotor and stator) to two-phase systems, whose axes are perpendicular to each other and no interaction takes place. Additionally the rotor will be referred to the stator winding.
2. Transformation of the steady stator winding and rotating rotor winding to an arbitrary system, rotating with angular velocity. Thereby the mutual inductances no longer depend on the rotor position.
3. Setting up the voltage equations for rotor and stator in the transformed system, rotating with arbitrary angular speed.
4. Determination of the torque from balance of power.

Afterwards the dynamic, power converter and the transient behavior of DC, induction and synchronous machine will be investigated both analytically as well as numerically.

1.4 Transformation from 3 to 2 phases

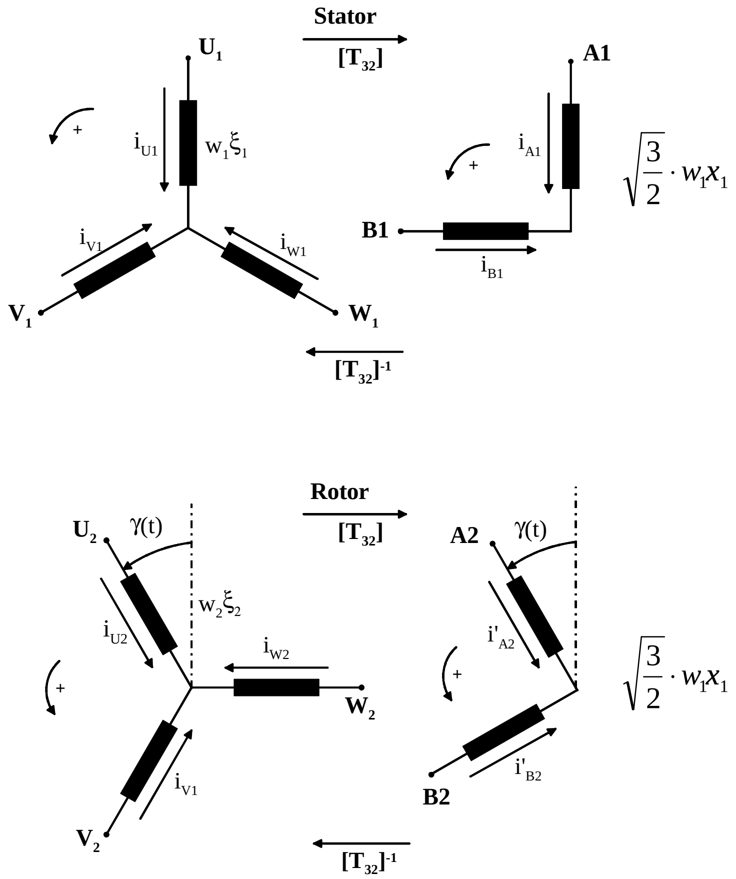


Fig. 2: three-to-two phase transformation

Conditions for rotating fields:

- three-phase winding: $p \cdot a_{mech} = a_{el} = \frac{2p}{3}$
- two-phase winding: $p \cdot a_{mech} = a_{el} = \frac{p}{2}$

For a power invariant transformation, equation 1.12 must apply:

$$S = 3 \cdot U_{3Ph} \cdot I_{3Ph} = 2 \cdot U_{2Ph} \cdot I_{2Ph} \tag{1.12}$$

According to this, there are different opportunities:

$$U_{2Ph} = \frac{3}{2} U_{3Ph}, \qquad I_{2Ph} = I_{3Ph} \tag{1.13}$$

$$U_{2Ph} = U_{3Ph}, \qquad I_{2Ph} = \frac{3}{2} I_{3Ph} \tag{1.14}$$

$$U_{2Ph} = \sqrt{\frac{3}{2}} \cdot U_{3Ph}, \qquad I_{2Ph} = \sqrt{\frac{3}{2}} \cdot I_{3Ph} \text{ (symmetrical)} \tag{1.15}$$

Which of these possibilities is best suitable?

If invariant transformation of resistances and inductances is demanded besides invariant transformation of power, only the symmetric option is possible, which will be shown by the following considerations:

$$R = r \cdot \frac{w \cdot l_m}{q_L} = r \cdot \frac{w^2 \cdot l_m \cdot 2 \cdot m}{A_{Cu}} \qquad \text{with } q_L = \frac{A_{Cu}}{2 \cdot m \cdot w} \tag{1.16}$$

$$L = \frac{m}{2} \cdot m_0 \cdot \left(\frac{w_1 x_1}{p} \right)^2 \cdot \frac{2}{p} \cdot \frac{l \cdot D}{d} \tag{1.17}$$

To conserve previous resistances and inductances, the number of windings results in:

$$\begin{aligned} & m \cdot (w \cdot x)^2 = const \\ & 3 \cdot (w \cdot x)^2_{3Ph} = 2 \cdot (w \cdot x)^2_{2Ph} \\ & (w \cdot x)_{2Ph} = \sqrt{\frac{3}{2}} \cdot (w \cdot x)_{3Ph} \end{aligned} \tag{1.18}$$

Therefore follows from the equality of the current linkage for the currents:

$$\begin{aligned} & \frac{3}{2} \cdot \frac{4}{p} \cdot \frac{(w \cdot x)_{3Ph}}{p} \cdot \sqrt{2} \cdot I_{3Ph} = \frac{2}{2} \cdot \frac{4}{p} \cdot \frac{(w \cdot x)_{2Ph}}{p} \cdot \sqrt{2} \cdot I_{2Ph} \\ & I_{2Ph} = \sqrt{\frac{3}{2}} \cdot I_{3Ph} \end{aligned} \tag{1.19}$$

This means equal air-gap induction and for the case of same geometric data: equal air-gap flux.

Therefore applies for the voltages:

$$\begin{aligned} & \frac{U_{2Ph}}{U_{3Ph}} = \frac{\frac{w}{\sqrt{2}} \cdot (w \cdot x)_{2Ph} \cdot f}{\frac{w}{\sqrt{2}} \cdot (w \cdot x)_{3Ph} \cdot f} \\ & \frac{U_{2Ph}}{U_{3Ph}} = \sqrt{\frac{3}{2}} \end{aligned} \tag{1.20}$$

The symmetrical transformation to a system with $\sqrt{\frac{3}{2}} \cdot (w \cdot x)$ is therefore power-, resistance- and inductance invariant and will be used below.

The conversion of the stator will be performed as follows:

If a three-phase system is intended to be replaced by a two phase system, it is advisable to set the axis of winding A so that it coincides with the orientation of axis U. The axis B is perpendicular to A. The turn ratio is:

$$(w_1 \cdot x_1)_{2Ph} = \sqrt{\frac{3}{2}} \cdot (w_1 \cdot x_1)_{3Ph} \, . \tag{1.21}$$

Both systems must have the same ampere turns across the air gap. Then the currents must be

- for A-axis:

$$w_1 x_1 \cdot \left(i_{u1} + i_{v1} \cdot \cos\frac{2p}{3} + i_{w1} \cdot \cos\frac{4p}{3} \right) = \sqrt{\frac{3}{2}} w_1 x_1 \cdot i_{A1} \tag{1.22}$$

$$\left(i_{u1} - \frac{i_{v1}}{2} - \frac{1}{2}\left(-i_{u1} - i_{v1} \right) \right) = \sqrt{\frac{3}{2}} \cdot i_{A1} \tag{1.23}$$

$$i_{A1} = \sqrt{\frac{3}{2}} \cdot i_{u1} \tag{1.24}$$

- for B-axis:

$$w_1 x_1 \cdot \left(i_{v1} \cdot \cos\frac{p}{6} + i_{w1} \cdot \cos\frac{5p}{6} \right) = \sqrt{\frac{3}{2}} w_1 x_1 \cdot i_{B1} \tag{1.25}$$

$$\frac{\sqrt{3}}{2} \cdot i_{v1} - \frac{\sqrt{3}}{2} \cdot i_{w1} = \sqrt{\frac{3}{2}} \cdot i_{B1} \tag{1.26}$$

$$\frac{i_{v1}}{2} - \frac{1}{2} \cdot \left(-i_{u1} - i_{v1} \right) = \frac{i_{B1}}{\sqrt{2}} \tag{1.27}$$

$$i_{B1} = \frac{i_{u1}}{\sqrt{2}} + i_{v1}\sqrt{2} \tag{1.28}$$

The transformation form of the three phase system to the equivalent two phase system can also be written in matrix form:

$$\begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} = \left[T_{32} \right] \cdot \begin{bmatrix} i_{u1} \\ i_{v1} \end{bmatrix} \quad \text{with } \left[T_{32} \right] = \begin{bmatrix} \sqrt{\frac{3}{2}} & 0 \\ \frac{1}{\sqrt{2}} & \sqrt{2} \end{bmatrix}. \tag{1.29}$$

The conversion of the voltages will be performed in a similar way, because the symmetric transformation was chosen:

$$\begin{bmatrix} u_{A1} \\ u_{B1} \end{bmatrix} = \left[T_{32} \right] \cdot \begin{bmatrix} u_{u1} \\ u_{v1} \end{bmatrix}. \tag{1.30}$$

Just as well a two phase system can be transformed to a three phase system by reverting the equations:

$$\begin{bmatrix} i_{u1} \\ i_{v1} \end{bmatrix} = \left[T_{32} \right]^{-1} \cdot \begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} \quad \text{with } \left[T_{32} \right]^{-1} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{1.31}$$

$$\begin{bmatrix} u_{u1} \\ u_{v1} \end{bmatrix} = \left[T_{32} \right]^{-1} \cdot \begin{bmatrix} u_{A1} \\ u_{B1} \end{bmatrix} \tag{1.32}$$

$$\text{whereas } \left[T_{32} \right] \cdot \left[T_{32} \right]^{-1} = \left[1 \right]. \tag{1.33}$$

The conversion of the rotor from three to two phases can be performed in analogy. Additionally the rotor will be translated to the number of stator turns per unit length, which is marked in the following with a dash " ' ".

$$ü = \frac{w_1 x_1}{w_2 x_2} \tag{1.34}$$

$$\begin{bmatrix} u'_{A2} \\ u'_{B2} \end{bmatrix} = [T_{32}] \cdot ü \cdot \begin{bmatrix} u_{u2} \\ u_{v2} \end{bmatrix} \tag{1.35}$$

$$\begin{bmatrix} i'_{A2} \\ i'_{B2} \end{bmatrix} = [T_{32}] \cdot \frac{1}{ü} \cdot \begin{bmatrix} i_{u2} \\ i_{v2} \end{bmatrix} \tag{1.36}$$

The conversion of the flux linkages of the stator and rotor winding takes place in the manner from three to two phases and vice versa.

$$\begin{bmatrix} \Psi_{A1} \\ \Psi_{B1} \end{bmatrix} = [T_{32}] \cdot \begin{bmatrix} \Psi_{u1} \\ \Psi_{v1} \end{bmatrix} \tag{1.37}$$

$$\begin{bmatrix} \Psi'_{A2} \\ \Psi'_{B2} \end{bmatrix} = [T_{32}] \cdot ü \cdot \begin{bmatrix} \Psi_{u2} \\ \Psi_{v2} \end{bmatrix} \tag{1.38}$$

1.5 Transformation of the 2 phase rotor and stator to an arbitrary revolving coordinate system

To obtain constant mutual inductances, e.g. for salient pole synchronous machines or for the application of specific control algorithms (e.g. field oriented control), rotor or stator of three phase machines must be transformed to stator or rotor. Sometimes it might be useful to transform the rotor or stator to a coordinate system to rotate with the air gap flux.

In the following a generic transformation from the inactive stator and the with $\frac{dg}{dt}$ rotating rotor to a coordinate system, rotating with arbitrary angular speed $\frac{da}{dt}$ is presented.

The equation system is then applicable to various machine types or can be chosen freely:

- $a = 0$ inactive coordinate system,
- $a = w \cdot t$ coordinate system rotating with rotor speed ($w = \frac{dg}{dt}$),
- $a = w_1 \cdot t$ coordinate system rotating with synchronous speed (line-frequency),
- $a = w_m \cdot t$ coordinate system rotating with air-gap flux (variable frequency).

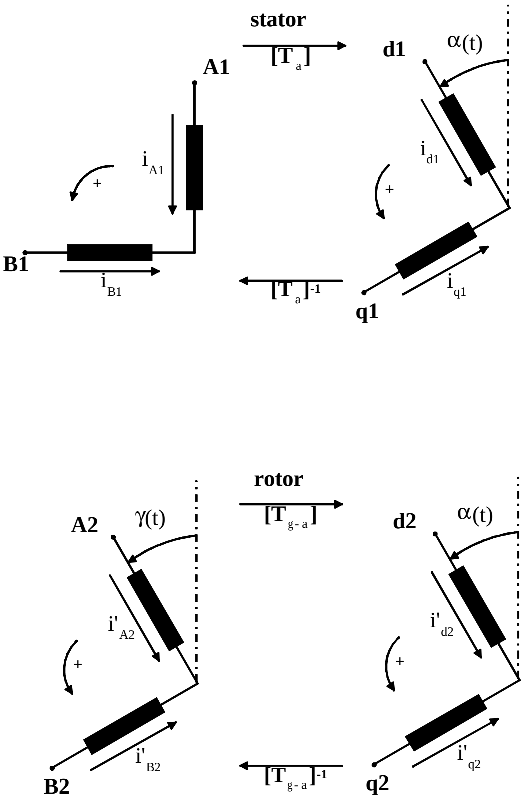


Fig. 3: transformation to revolving coordinate system

Therefore equations for rotating Cartesian coordinate systems in a plane are used:

$$x^{*} = x \cdot \cos a + y \cdot \sin a \qquad (1.43)$$

$$y^{*} = -x \cdot \sin a + y \cdot \cos a \qquad (1.43)$$

Carried forward currents and voltages may be written in matrix form:

$$\begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} = [T_a] \cdot \begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} \qquad (1.43)$$

$$\begin{bmatrix} u_{d1} \\ u_{q1} \end{bmatrix} = [T_a] \cdot \begin{bmatrix} u_{A1} \\ u_{B1} \end{bmatrix}, \qquad (1.43)$$

with transformation matrix of stator to a revolving system

$$[T_a] = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix}. \qquad (1.43)$$

While transforming the rotor to the rotating system care has to be taken since both system are moving relative to each other with the difference angel $g - a$.

$$\begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} = [T_{g-a}] \cdot \begin{bmatrix} i'_{A2} \\ i'_{B2} \end{bmatrix} \qquad (1.44)$$

$$\begin{bmatrix} u'_{d2} \\ u'_{q2} \end{bmatrix} = [T_{g-a}] \cdot \begin{bmatrix} u'_{A2} \\ u'_{B2} \end{bmatrix} \qquad (1.45)$$

with the transformation matrix of the rotor to a revolving system

$$[T_{g-a}] = \begin{bmatrix} \cos(g - a) & -\sin(g - a) \\ \sin(g - a) & \cos(g - a) \end{bmatrix}. \qquad (1.46)$$

The reverse transformation is performed with the inverse matrices:

$$[T_a]^{-1} = \begin{bmatrix} +\cos a & -\sin a \\ +\sin a & +\cos a \end{bmatrix} \qquad (1.47)$$

$$[T_{g-a}]^{-1} = \begin{bmatrix} +\cos(g - a) & +\sin(g - a) \\ -\sin(g - a) & +\cos(g - a) \end{bmatrix} \qquad (1.48)$$

$$\begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} = [T_a]^{-1} \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} \qquad (1.49)$$

$$\begin{bmatrix} u_{A1} \\ u_{B1} \end{bmatrix} = [T_a]^{-1} \cdot \begin{bmatrix} u_{d1} \\ u_{q1} \end{bmatrix} \qquad (1.50)$$

$$\begin{bmatrix} i'_{A2} \\ i'_{B2} \end{bmatrix} = \left[T_{\gamma-\alpha}\right]^{-1} \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} \tag{1.51}$$

$$\begin{bmatrix} u'_{A2} \\ u'_{B2} \end{bmatrix} = \left[T_{\gamma-\alpha}\right]^{-1} \cdot \begin{bmatrix} u'_{d2} \\ u'_{q2} \end{bmatrix} \tag{1.52}$$

This transformation is also invariant of power, because the matrices are orthogonal, i.e.

$$\left[T_{\alpha}\right]^{T} = \begin{bmatrix} +\cos\alpha & -\sin\alpha \\ +\sin\alpha & +\cos\alpha \end{bmatrix} = \left[T_{\alpha}\right]^{-1} \tag{1.53}$$

$$\left[T_{\gamma-\alpha}\right]^{T} = \begin{bmatrix} +\cos(\gamma-\alpha) & +\sin(\gamma-\alpha) \\ -\sin(\gamma-\alpha) & +\cos(\gamma-\alpha) \end{bmatrix} = \left[T_{\gamma-\alpha}\right]^{-1} \tag{1.54}$$

Since the same transformation is applied to voltages and currents, the quotient is constant, i.e. the transformation is invariant to inductances and resistances.
The flux linkages in the arbitrary rotating system can now be written as

$$\begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} = L_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + L_h \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} \tag{1.55}$$

$$\begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} = L'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + L_h \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} \tag{1.56}$$

After conversion to the same number of turns, rotor resistance and inductance are given by:

$$R'_2 = \ddot{u}^2 \cdot R_2 \tag{1.57}$$

$$L_h = \frac{3}{2} \cdot L_{hw} = \frac{3}{2} \cdot \ddot{u} \cdot M_{\mathrm{W}} \tag{1.58}$$

The total inductance is

$$L_1 = L_{1s} + L_h \tag{1.59}$$

$$L'_2 = L'_{2s} + L_h = \ddot{u}^2 \cdot L_2 \tag{1.60}$$

and the leakage factors ensue to

$$\sigma = 1 - \frac{L_h^2}{L_1 \cdot L'_2} = 1 - \frac{1}{(1+\sigma_1)\cdot(1+\sigma_2)} \tag{1.61}$$

$$\sigma_1 = \frac{L_{1s}}{L_h}, \qquad \sigma_2 = \frac{L'_{2s}}{L_h}. \tag{1.62 a, b}$$

It is remarkable, that mutual inductances do not appear in the arbitrary rotating system any longer. This is caused by the fact that transformed rotor and stator windings rotate with the same angular speed. Only the axes d1-d2 and q1-q2 are magnetically coupled.

1.6 Voltage equations in the arbitrary system

The transformation is to be performed in two steps:

1. Transformation from the three phase to the two phase system - non connected star point:

$$\begin{bmatrix} u_{A1} \\ u_{B1} \end{bmatrix} = [T_{32}] \cdot \begin{bmatrix} u_{u1} \\ u_{v1} \end{bmatrix} = R_1 \cdot [T_{32}] \cdot \begin{bmatrix} i_{u1} \\ i_{v1} \end{bmatrix} + \frac{d}{dt}[T_{32}] \cdot \begin{bmatrix} \Psi_{u1} \\ \Psi_{v1} \end{bmatrix} = R_1 \cdot \begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_{A1} \\ \Psi_{B1} \end{bmatrix} \quad (1.63)$$

$$\begin{bmatrix} u'_{A2} \\ u'_{B2} \end{bmatrix} = [T_{32}] \cdot \begin{bmatrix} u'_{u2} \\ u'_{v2} \end{bmatrix} = R'_2 \cdot [T_{32}] \cdot \begin{bmatrix} i'_{u2} \\ i'_{v2} \end{bmatrix} + \frac{d}{dt}[T_{32}] \cdot \begin{bmatrix} \Psi'_{u2} \\ \Psi'_{v2} \end{bmatrix} = R_2 \cdot \begin{bmatrix} i'_{A2} \\ i'_{B2} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi'_{A2} \\ \Psi'_{B2} \end{bmatrix} \quad (1.64)$$

with $[T_{32}]$ being constant and independent of *t*.

2. Transformation of the inactive stator and the rotating rotor to an arbitrary rotating coordinate system:

$$\begin{aligned}
\begin{bmatrix} u_{d1} \\ u_{q1} \end{bmatrix} &= [T_a] \cdot \begin{bmatrix} u_{A1} \\ u_{B1} \end{bmatrix} = R_1 \cdot [T_a] \cdot \begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} + [T_a] \cdot \frac{d}{dt}\left\{ [T_a]^{-1} \cdot \begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} \right\} \\
&= R_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + [T_a] \cdot \left\{ \frac{d[T_a]^{-1}}{dt} \cdot \begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} + [T_a]^{-1} \cdot \frac{d}{dt}\begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} \right\} \\
&= R_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} + \frac{da}{dt} \cdot \begin{bmatrix} -\Psi_{q1} \\ +\Psi_{d1} \end{bmatrix} \qquad (1.65)
\end{aligned}$$

Keep in mind, that in this case $[T_a]$ as well as $[T_{g-a}]$ are not independent of *t*.

$$\begin{aligned}
\begin{bmatrix} u'_{d2} \\ u'_{q2} \end{bmatrix} &= [T_{g-a}] \cdot \begin{bmatrix} u'_{A2} \\ u'_{B2} \end{bmatrix} = R'_2 \cdot [T_{g-a}] \cdot \begin{bmatrix} i'_{A2} \\ i'_{B2} \end{bmatrix} + [T_{g-a}] \cdot \frac{d}{dt}\left\{ [T_{g-a}]^{-1} \cdot \begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} \right\} \\
&= R'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + [T_{g-a}] \cdot \left\{ \frac{d[T_{g-a}]^{-1}}{dt} \cdot \begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} + [T_{g-a}]^{-1} \cdot \frac{d}{dt}\begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} \right\} \\
&= R'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} + \frac{d(g-a)}{dt} \cdot \begin{bmatrix} +\Psi'_{q2} \\ -\Psi'_{d2} \end{bmatrix} \qquad (1.66)
\end{aligned}$$

The voltage equations consist of the following aspects:

- voltage drop at the ohmic resistance
- temporal alteration of the flux linkage
- rotary voltages caused by $\frac{da}{dt}$ or. $\frac{d(g-a)}{dt}$

It is required to take a closer look at the following special cases and distinguish between:

$a(t)=0$: transformation to a stationary system: stator

$$\begin{bmatrix} u_{d1} \\ u_{q1} \end{bmatrix} = R_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} \tag{1.67}$$

$$\begin{bmatrix} u'_{d2} \\ u'_{q2} \end{bmatrix} = R'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} + \frac{dg}{dt} \cdot \begin{bmatrix} +\Psi'_{q2} \\ -\Psi'_{d2} \end{bmatrix} \tag{1.68}$$

$a(t)=g(t)$: transformation to a system, rotating with $g(t)$: rotor

$$\begin{bmatrix} u_{d1} \\ u_{q1} \end{bmatrix} = R_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} + \frac{dg}{dt} \cdot \begin{bmatrix} -\Psi_{q1} \\ +\Psi_{d1} \end{bmatrix} \tag{1.69}$$

$$\begin{bmatrix} u'_{d2} \\ u'_{q2} \end{bmatrix} = R'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} \tag{1.70}$$

1.7 Balance of power and torque

To determine the torque in an arbitrary rotating system, the balance of power is to be set up. The following matrices are therefore defined:

$$[i] = \begin{bmatrix} i_{d1} \\ i_{q1} \\ i'_{d2} \\ i'_{q2} \end{bmatrix} \quad [u] = \begin{bmatrix} u_{d1} \\ u_{q1} \\ u'_{d2} \\ u'_{q2} \end{bmatrix} \quad [\Psi] = \begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \\ \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} \qquad [w \cdot \Psi] = \begin{bmatrix} -\Psi_{q1} \cdot \frac{da}{dt} \\ +\Psi_{d1} \cdot \frac{da}{dt} \\ +\Psi'_{q2} \cdot \frac{d(g-a)}{dt} \\ -\Psi'_{d2} \cdot \frac{d(g-a)}{dt} \end{bmatrix} \tag{1.71}$$

So that the power input appears as:

$$P_{auf} = [i]^T \cdot [u] = \underbrace{[i]^T \cdot R \cdot [i]}_{P_V} + \underbrace{[i]^T \cdot \frac{d}{dt}[\Psi]}_{\frac{dW_m}{dt}} + \underbrace{[i]^T \cdot [w \cdot \Psi]}_{P_{mech}}, \tag{1.72}$$

where P_V represents the ohmic losses in the winding resistances, $\frac{dW_m}{dt}$ the change of the magnetic energy and P_{mech} the mechanical power of the total system:

$$P_{mech} = M_{el} \cdot \Omega = \frac{M_{el} \cdot w}{p} = \frac{M_{el} \cdot \frac{dg}{dt}}{p}. \tag{1.73}$$

Torque derives from

$$\begin{aligned} M_{el} &= \frac{p}{\frac{dg}{dt}}\left\{i_{d1}\cdot\frac{da}{dt}\cdot\left(-\Psi_{q1}\right)+i_{q1}\cdot\frac{da}{dt}\cdot\Psi_{d1}+i_{d2}^{'}\cdot\frac{d(g-a)}{dt}\cdot\Psi_{q2}^{'}+i_{q2}^{'}\cdot\frac{d(g-a)}{dt}\cdot\left(-\Psi_{d2}^{'}\right)\right\} \\ &= \frac{p}{\frac{dg}{dt}}\left\{\left(i_{q1}\cdot\Psi_{d1}-i_{d1}\cdot\Psi_{q1}\right)\frac{da}{dt}+\left(i_{d2}^{'}\cdot\Psi_{q2}^{'}-i_{q2}^{'}\cdot\Psi_{d2}^{'}\right)\cdot\frac{d(g-a)}{dt}\right\} \\ &= p\cdot L_{1h}\cdot\left(i_{q1}\cdot i_{d2}^{'}-i_{d1}\cdot i_{q2}^{'}\right). \end{aligned} \tag{1.74}$$

The equilibrium of torque is valid:

$$M_{el} = M_W + J\cdot\frac{d\Omega}{dt} = M_W + \frac{J}{p}\cdot\frac{d^2g}{dt^2} \tag{1.75}$$

$$\Omega = 2\cdot p\cdot n = \frac{w}{p} = \frac{1}{p}\cdot\frac{dg}{dt} \tag{1.76}$$

Two different cases need to be distinguished:

- $a(t) = 0$: steady coordinate system

$$M_{el} = p\cdot\left(i_{d2}^{'}\cdot\Psi_{q2}^{'}-i_{q2}^{'}\cdot\Psi_{d2}^{'}\right) \tag{1.77}$$

- $a(t) = g(t)$: with the rotor rotating coordinate system

$$M_{el} = p\cdot\left(i_{q1}\cdot\Psi_{d1}-i_{d1}\cdot\Psi_{q1}\right) \tag{1.78}$$

1.8 Compilation of the equations of the direct and quadrature axis theory

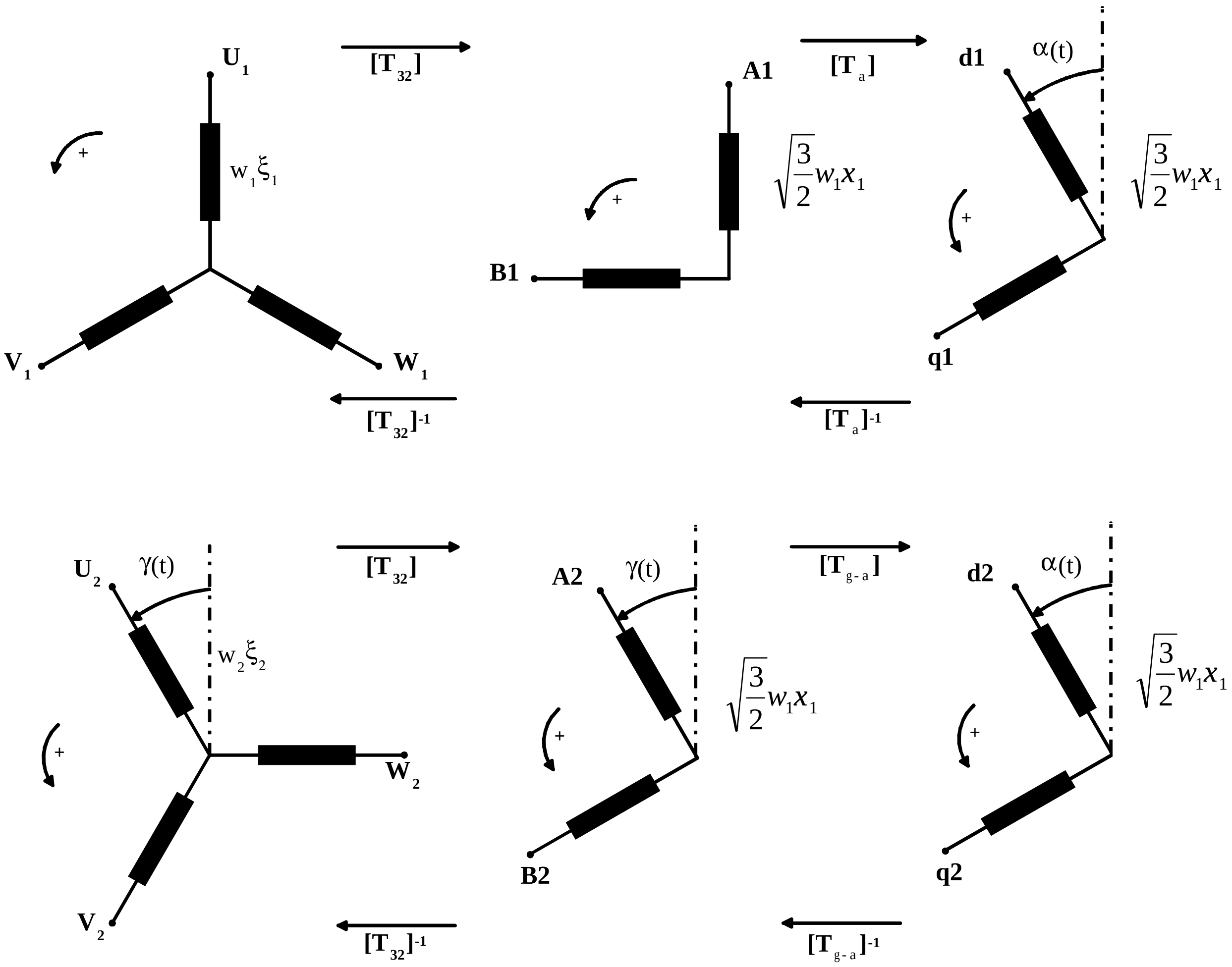


Fig. 4: transformation of revolving 2-axis coordinate system

$$\begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} = [T_{32}] \cdot \begin{bmatrix} i_{u1} \\ i_{v1} \end{bmatrix} \qquad \begin{bmatrix} i'_{A2} \\ i'_{B2} \end{bmatrix} = [T_{32}] \cdot \begin{bmatrix} \mathrm{i}'_{\mathrm{u2}} \\ \mathrm{i}'_{\mathrm{v2}} \end{bmatrix} \tag{1.79}$$

$$\begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} = [T_a] \cdot \begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} \qquad \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} = [T_{g-a}] \cdot \begin{bmatrix} \mathrm{i}'_{\mathrm{A2}} \\ \mathrm{i}'_{\mathrm{B2}} \end{bmatrix} \tag{1.80}$$

$$[T_{32}] = \begin{bmatrix} \sqrt{\frac{3}{2}} & 0 \\ \frac{1}{\sqrt{2}} & \sqrt{2} \end{bmatrix} \qquad [T_a] = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} \quad [T_{g-a}] = \begin{bmatrix} \cos(g-a) & -\sin(g-a) \\ \sin(g-a) & \cos(g-a) \end{bmatrix} \tag{1.81}$$

$$\begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} = L_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + L_h \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} \qquad \begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} = L'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + L_h \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} \tag{1.82}$$

$$\begin{bmatrix} u_{d1} \\ u_{q1} \end{bmatrix} = R_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} + \frac{da}{dt} \cdot \begin{bmatrix} -\Psi_{q1} \\ +\Psi_{d1} \end{bmatrix} \tag{1.83}$$

$$\begin{bmatrix} u'_{d2} \\ u'_{q2} \end{bmatrix} = R'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} + \frac{d(g-a)}{dt} \cdot \begin{bmatrix} +\Psi'_{q2} \\ -\Psi'_{d2} \end{bmatrix} \tag{1.84}$$

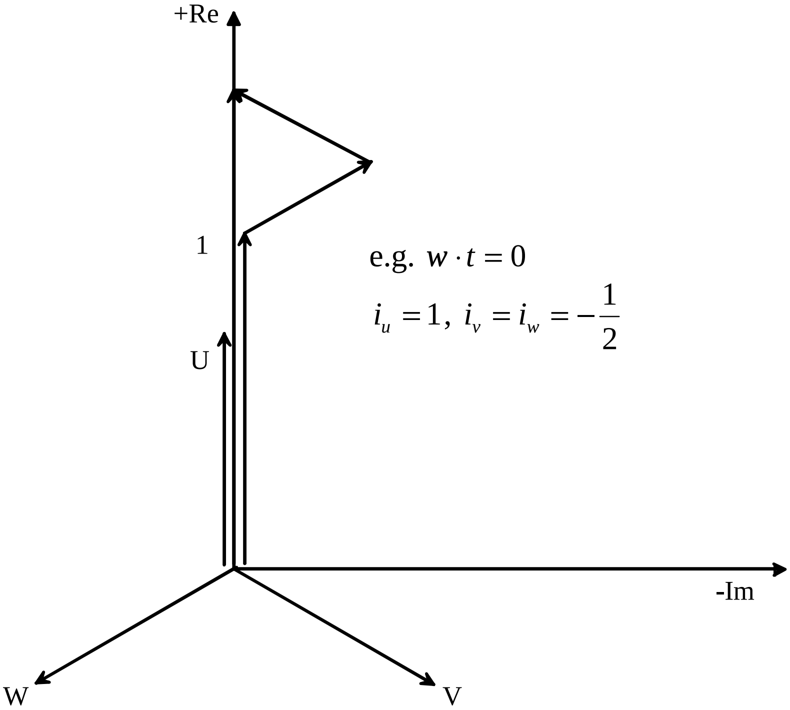
$$M_{el} = p \cdot L_{1h} \cdot \left(i_{q1} \cdot i'_{d2} - i_{d1} \cdot i'_{q2} \right) = \frac{J}{p} \cdot \frac{d^2 g}{dt^2} + M_W \tag{1.85}$$

1.9 Space vectors

Complex space vectors are often used in common in the literature. In the complex area a rotating field can be represented by a rotating space vector with the angular velocity ω. The position of the space vector describes the instantaneous maximum and the quantity of the space vector describes the amplitude of the rotating field. The complex space vector is constructed using the instantaneous values of the three phases, for example of the currents:

$$\underline{i}(t) = \frac{2}{3} \cdot \left(i_u(t) + \underline{a} \cdot i_v(t) + \underline{a}^2 \cdot i_w(t) \right) \tag{1.86}$$

The operator $\underline{a} = e^{j \cdot \frac{2p}{3}}$ represents a spatial displacement of 120°. With the factor $\frac{2}{3}$ the amplitude of the space vector is adjusted to the physical magnitude. The projection on the particular winding axis results the instantaneous value of the particular phase current.



As well as the currents, the voltages and the flux linkages can be defined using space vectors.

Fig. 5: space vector illustration

If the dynamic equations of a rotating field machine are represented with complex space vectors in a coordinate system, which is rotating with angular speed $a(t)$, the following set of equations is achieved, if the rotor is converted to the number of the stator windings:

$$\underline{u}_S = R_1 \cdot \underline{i}_S + \frac{d\underline{\Psi}_S}{dt} + j \cdot \frac{da}{dt} \cdot \underline{\Psi}_S \tag{1.87}$$

$$\underline{u}'_R = R'_2 \cdot \underline{i}'_R + \frac{d\underline{\Psi}'_R}{dt} + j \cdot \frac{d(g-a)}{dt} \cdot \underline{\Psi}'_R \tag{1.88}$$

$$\underline{\Psi}_S = L_1 \cdot \underline{i}_S + L_{1h} \cdot \underline{i}'_R \tag{1.89}$$

$$\underline{\Psi}'_R = L_2 \cdot \underline{i}'_R + L_{1h} \cdot \underline{i}_S \tag{1.90}$$

$$M_{el} = \frac{3}{2} p \,\mathrm{Im}\left(\underline{\Psi}'_R \cdot \underline{i}'^*_R\right) + \frac{J}{p} \cdot \frac{dw}{dt} \qquad (* \mathrel{\hat{=}} \text{conjugate complex}) \tag{1.91}$$

If the complex space vectors are split up into their components

$$\underline{u}_S = u_{q1} - j \cdot u_{d1} \tag{1.92}$$

$$\underline{u}'_R = u'_{q2} - j \cdot u'_{d2} \tag{1.93}$$

$$\underline{i}_S = i_{q1} - j \cdot i_{d1} \tag{1.94}$$

$$\underline{i}'_R = i'_{q2} - j \cdot i'_{d2} \tag{1.95}$$

$$\underline{\Psi}_S = \Psi_{q1} - j \cdot \Psi_{d1} \tag{1.96}$$

$$\underline{\Psi}'_R = \Psi'_{q2} - j \cdot \Psi'_{d2} \tag{1.97}$$

with “d” representing the negative imaginary axis and “q” representing the positive real axis, a set of equations is gained, which is, except of the factor in the torque equation, similar to the set of equations of the two-axis theory.

$$\begin{bmatrix} u_{d1} \\ u_{q1} \end{bmatrix} = R_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} + \frac{da}{dt} \cdot \begin{bmatrix} -\Psi_{q1} \\ +\Psi_{d1} \end{bmatrix} \tag{1.98}$$

$$\begin{bmatrix} u'_{d2} \\ u'_{q2} \end{bmatrix} = R'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} + \frac{d(g-a)}{dt} \cdot \begin{bmatrix} +\Psi'_{q2} \\ -\Psi'_{d2} \end{bmatrix} \tag{1.99}$$

$$M_{el} = \frac{3}{2} p \cdot \left(\Psi'_{q2} \cdot i'_{d2} - \Psi'_{d2} \cdot i'_{q2}\right) + \frac{J}{p} \cdot \frac{dw}{dt} \tag{1.100}$$

The factor $\frac{3}{2}$ in the torque equation shows, that the transformation is not invariant of power. Instead of that the space vectors should be defined as follows (for example for the current):

$$\underline{i} = \sqrt{\frac{2}{3}} \cdot \left(i_u + \underline{a} \cdot i_v + \underline{a}^2 \cdot i_w\right) \tag{1.101}$$

Thus the according space vectors would loose clearness.

2 DC machine

2.1 Basics

Because of their easy control and because of economic reasons DC machines in combination with thyristor converters or transistor amplifiers are today still often used as high dynamic speed-variable drives in the small and middle power range.

The essential disadvantage is the commutator. Size and dynamic are limited by the mechanical commutation. The current transfer with brushes has a high rate of wear and requires maintenance.
But because of the mechanical commutation with the commutator, the armature current linkage and the excitation field are always oriented in an optimal way for the generation of the torque. With constant excitation there is a linear correlation between torque and current. Thus a rapid and exact speed- and position control is possible.

The following pictures show the basic structure of controlled DC machines (For the simplification machines with only two poles are shown).

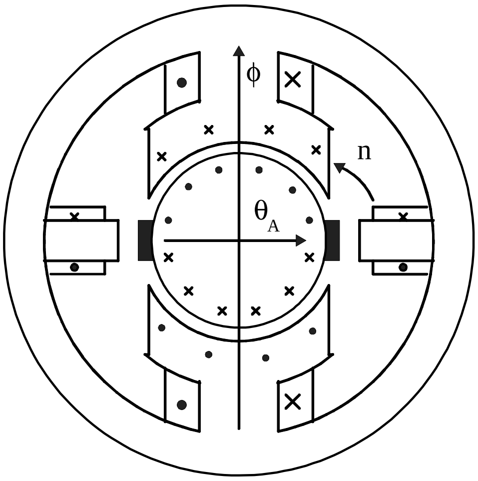


Fig. 6: DC machine

Fig. 5 shows a large machine in medium power range with some 100 kW for handling applications. The machine is equipped with commutation poles and a compensating-field winding, to improve the commutation and to suppress the armature reaction. The rotor and the field frame are made of laminated steel, to enable rapid changes of the magnetic field.

Fig. 6 sketches a cylindrical servomotor with radial field, which is typically used in the range of some kW for rapid positioning drives. The usage of rare-earth permanent magnets and a slim-line type rotor are benefiting because of dynamic reasons. With permanent magnets in the stator, the machine can only be controlled in the armature circuit.

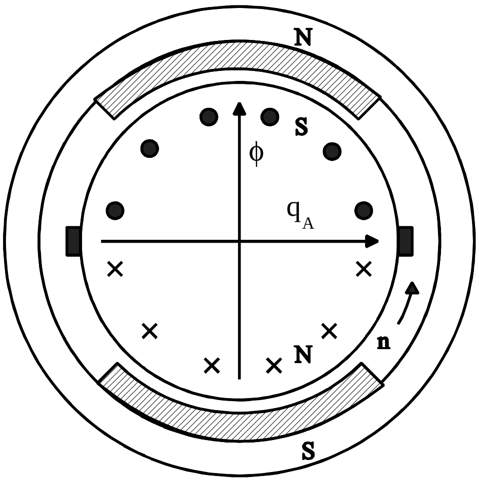


Fig. 7: DC machine, servo motor

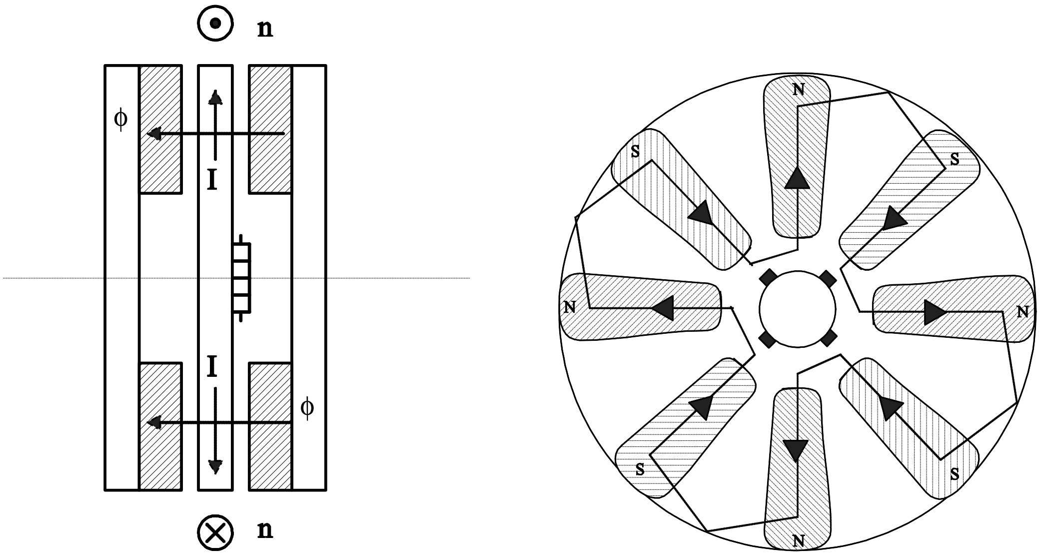
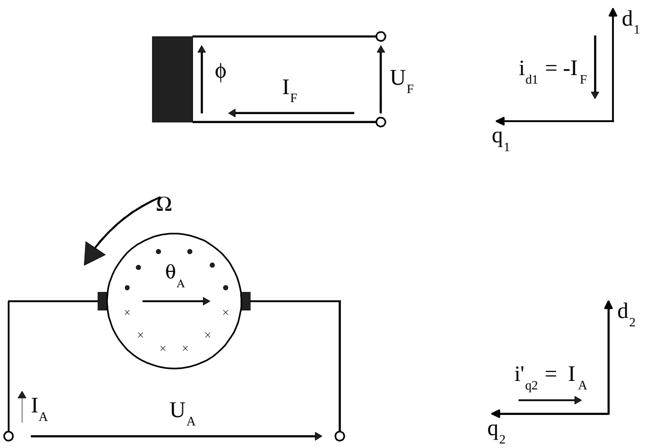


Fig. 8, 9: DC machine, disc type rotor (high dynamic)

Fig. 7 also illustrates a high-dynamic DC servomotor in the power range of some kW for robotic device applications. The motor has an axial field and a double-side stator with AlNiCo –magnets. The ironless disc-type rotor is made of distributed wires or punched segments.

2.2 Dynamic set of equations

The dynamic set of equations of DC machines can be directly deduced from the two axis theory. Variables need to be set to $a = 0$, i.e. $w_k = 0$, regarding:



The d1-axis corresponds to the excitation field.

The q2-axis is the axis of the armature quadrature-axis field.

The polarity of the excitation field is reversed, to turn the anti-clockwise rotation positive. There are no windings q1 and d2.

Fig. 10: two-axis theory applied on DC machine

The mechanical angular speed ensues to:

$$\frac{dg}{dt} = w = p\Omega \tag{2.1}$$

It has to be considered, that the rotor is converted to the number of the stator windings, which is not usual for DC machines. Therefore it has to be cancelled.

The following substitutions apply:

$$u_{d1} = -U_F\,, \quad i_{d1} = -I_F\,, \quad R_1 = R_F\,, \quad L_1 = L_F \qquad \text{(2.2 a-d)}$$

$$u_{q1} = 0\,,\ i_{q1} = 0 \tag{2.3}$$

$$u'_{d2} = 0\,,\ i'_{d2} = 0 \tag{2.4}$$

$$u'_{q2} = \ddot{u} \cdot U_A\,, \quad i'_{q2} = \frac{I_A}{\ddot{u}}\,, \quad R'_2 = \ddot{u}^2 \cdot R_A\,, \quad L'_2 = \ddot{u}^2 \cdot L_A \qquad \text{(2.5 a-d)}$$

$$\Psi_{d1} = -L_F \cdot I_F \tag{2.6}$$

$$\Psi_{q1} = L_h \cdot \frac{I_A}{\ddot{u}} \tag{2.7}$$

$$\Psi'_{d2} = -L_h \cdot I_F \tag{2.8}$$

$$\Psi'_{q2} = \ddot{u}^2 \cdot L_A \cdot \frac{I_A}{\ddot{u}} \tag{2.9}$$

Therefore the following equations are achieved:

$$-U_F = -R_F \cdot I_F - L_F \cdot \frac{dI_F}{dt} \tag{2.10}$$

$$\ddot{u} \cdot U_A = \ddot{u}^2 \cdot R_A \cdot \frac{I_A}{\ddot{u}} + \ddot{u} \cdot L_A \cdot \frac{dI_A}{dt} - p \cdot \Omega \cdot \left(-L_h \cdot I_F\right) \Bigg| \cdot \frac{1}{\ddot{u}} \tag{2.11}$$

$$M_{el} = p \cdot L_h \cdot \left(0 - \left(-I_F\right) \cdot \frac{I_A}{\ddot{u}}\right) = \frac{J}{p} \cdot p \cdot \frac{d\Omega}{dt} + M_W \tag{2.12}$$

With the mutual inductance between excitation- and armature winding $M = \frac{L_h}{\ddot{u}}$ three differential equations for DC machines are gained, which describe the dynamic behaviour:

$$U_F = R_F \cdot I_F + L_F \cdot \frac{dI_F}{dt} \tag{2.13}$$

$$U_A = R_A \cdot I_A + L_A \cdot \frac{dI_A}{dt} + p \cdot M \cdot I_F \cdot \Omega \tag{2.14}$$

$$M_{el} = p \cdot M \cdot I_F \cdot I_A = J \cdot \frac{d\Omega}{dt} + M_W \tag{2.15}$$

The saturation of the excitation circuit is neglected in the considered case.
In analogy to the lecture Electrical Machines I:

$$p \cdot M \cdot I_F = c \cdot f \tag{2.16}$$

$$c = \frac{k}{2p} \tag{2.17}$$

$$p \cdot M \cdot I_F \cdot \Omega = c \cdot f \cdot \Omega = \frac{k \cdot f}{2p} \cdot 2p \cdot n = k \cdot f \cdot n = U_i \tag{2.18}$$

$$p \cdot M \cdot I_F \cdot I_A = c \cdot f \cdot I_A = M_{el} \tag{2.19}$$

It is obvious from the dynamic set of equations, that not only the variables for the steady-state operation but additionally the energy stores L_F, L_A and J need to be taken into consideration.
Besides the saturation, the mechanical friction and the voltage drop at the brushes in the following paragraphs is neglected.
Fig. 11 shows the equivalent circuit diagram of the DC machine in dynamic operation.

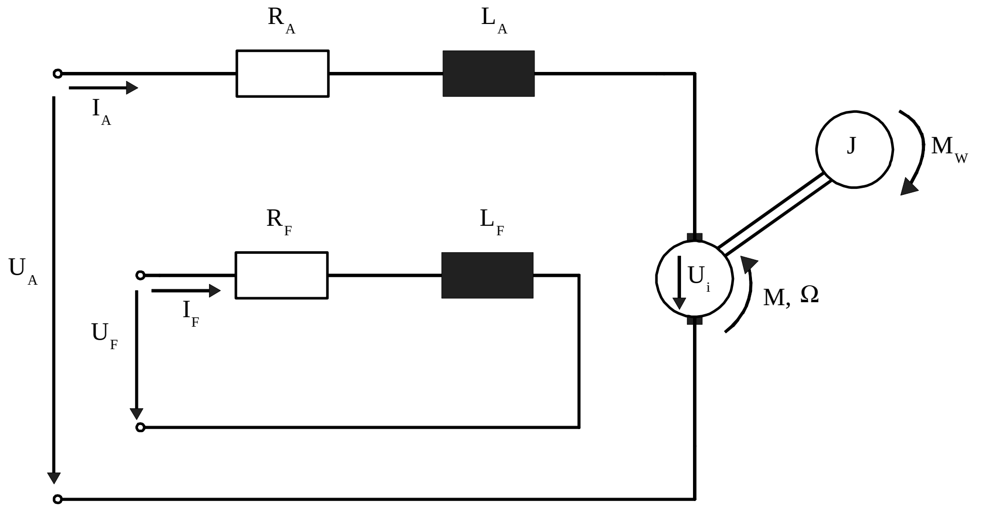


Fig. 11: DC machine, equivalent circuit diagram for dynamic operation

The set of equations needs to be converted into the state formulation. To simplify the following considerations, it is normalized on rated (=nominal) values:

$$L_F \cdot \frac{dI_F}{dt} = U_F - R_F \cdot I_F \qquad \Big| \cdot \frac{1}{R_F} \cdot \frac{1}{I_{FN}} \tag{2.20}$$

$$L_A \cdot \frac{dI_A}{dt} = U_A - R_A \cdot I_A - p \cdot M \cdot I_F \cdot \Omega \qquad \Big| \cdot \frac{1}{R_A} \cdot \frac{1}{I_{AN}} \tag{2.21}$$

$$J \cdot \frac{d\Omega}{dt} = p \cdot M \cdot I_F \cdot I_A - M_W \qquad \Big| \cdot \frac{1}{M_N} \tag{2.22}$$

Normalized values are also called “per-unit values”. For this purpose lower case letters are used:

$$\begin{aligned} &\frac{U_A}{U_{AN}} = u_A, \quad \frac{I_A}{I_{AN}} = i_A, \quad \frac{R_A \cdot I_{AN}}{U_{AN}} = r_A \\ &\frac{U_F}{U_{FN}} = u_F, \quad \frac{I_F}{I_{FN}} = i_F, \quad \frac{R_F \cdot I_{FN}}{U_{FN}} = r_F \\ &\frac{\Omega}{\Omega_0} = n, \quad \frac{M_W}{M_N} = m_w \end{aligned} \tag{2.23 a-h}$$

nominal values:

$$U_N = p \cdot M \cdot I_{FN} \cdot \Omega_0 \tag{2.24}$$

$$M_N = p \cdot M \cdot I_{FN} \cdot I_{AN} \tag{2.25}$$

time constants:

$$T_F = \frac{L_F}{R_F} \tag{2.26}$$

$$T_A = \frac{L_A}{R_A} \tag{2.27}$$

$$T_J = \frac{J \cdot \Omega_0}{M_N} \tag{2.28}$$

Referenced set of equations of the DC machine in state formulation:

$$T_F \cdot \frac{di_F}{dt} = \frac{u_F}{r_F} - i_F \tag{2.29}$$

$$T_A \cdot \frac{di_A}{dt} = \frac{1}{r_A} \cdot \left(u_A - \underbrace{i_F \cdot n}_{u_i} \right) - i_A \tag{2.30}$$

$$T_J \cdot \frac{dn}{dt} = \underbrace{i_F \cdot i_A}_{m_{el}} - m_w \tag{2.31}$$

- Input- or excitation variables: u_F, u_A, m_W
- State variables: i_F, i_A, n
- Machine parameters: T_F, T_A, T_J, r_A, r_F

The according structure diagram for the three differential equations, describing the state of the energy stores is depicted in Fig. 9:

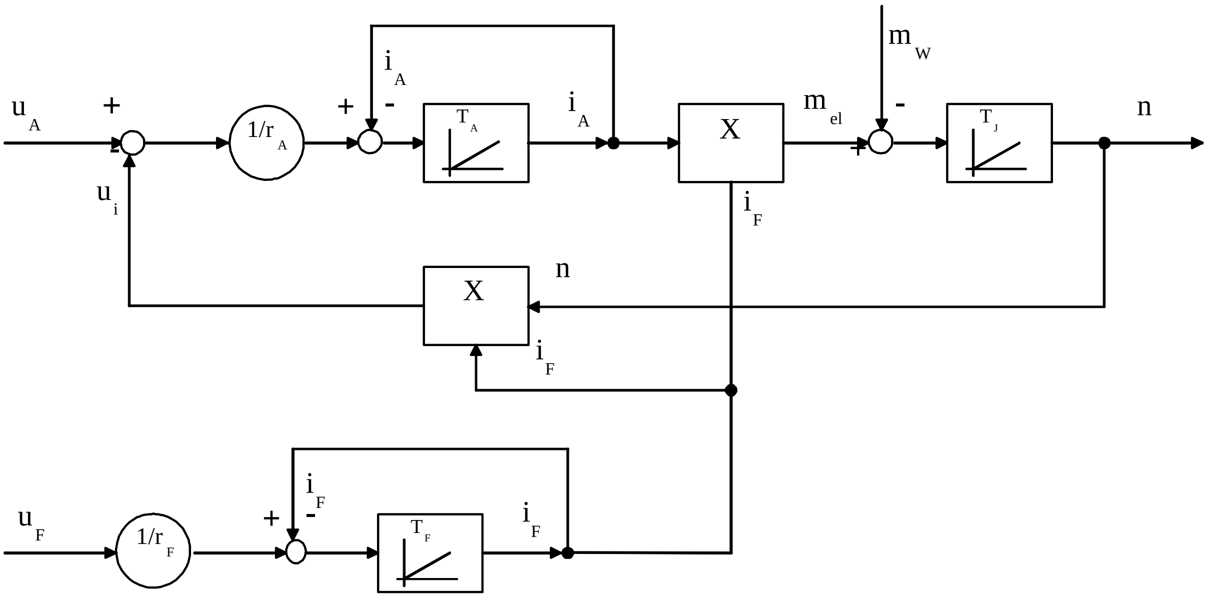


Fig. 12: equation set, structure diagram

Because of the products $i_A \cdot i_F$ and $n \cdot i_F$ the set of equations is not linear and can only be solved with numerical methods using computers.

2.3 Separately excited DC machine

In many cases speed-variable DC machines are operated with constant excitation or with permanent-magnet excitation. Thus torque and speed can only be influenced by adjusting the armature voltage. In this case $I_F = I_{FN} = const$ and the structure diagram of the motor simplifies a lot using $i_F = 1$, as shown in Fig. 13.

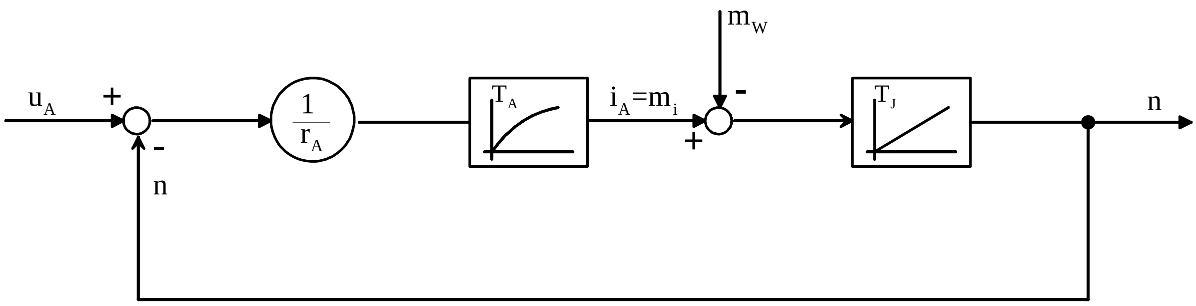


Fig. 13: DC machine, structure diagram

The set of equation now consists of only two equations being linear. Therefore it can be solved analytically:

$$T_A \cdot \frac{di_A}{dt} = \frac{u_A - n}{r_A} - i_A \tag{2.32}$$

$$T_J \cdot \frac{dn}{dt} = i_A - m_W \tag{2.33}$$

The structure diagram of DC machines is obtained using Laplace-transformation. This is the common display format in control engineering. In order to differ from the appearance of previous equations, upper case letters are used for Laplace-transformation.

$$T_A \cdot sI_A(s) + I_A(s) = \frac{U_A(s) - N(s)}{r_A} \tag{2.34}$$

$$T_J \cdot sN(s) = I_A(s) - M_W(s) \tag{2.35}$$

$$I_A(s) = \frac{U_A(s) - N(s)}{r_A \cdot (1 + sT_A)} \tag{2.36}$$

$$N(s) = \frac{I_A(s) - M_W(s)}{sT_J} \tag{2.37}$$

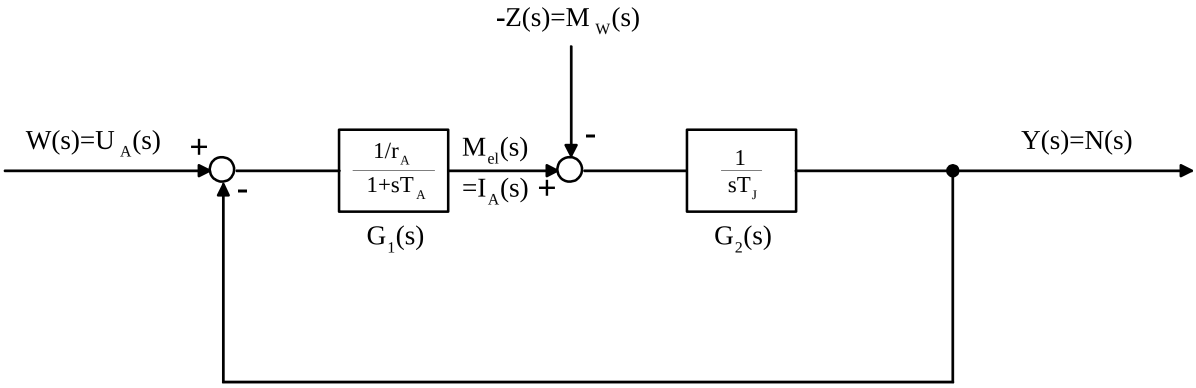


Fig. 14: DC machine, structure diagram due to Laplace transformation

The speed of the DC machine states the output variable, which is controlled by the armature voltage. So the armature voltage is the reference input variable. The load torque is the disturbance. The following correlation applies:

$$Y = \frac{G_1 G_2}{1+G_1 G_2} \cdot W + \frac{G_2}{1+G_1 G_2} \cdot Z \tag{2.38}$$

1.) <u>Response to setpoint changes:</u>

The response to set-point changes (this is the speed variation of the DC machine if the armature voltage changes) is obtained for $Z \mathrel{\hat{=}} M_W = 0$:

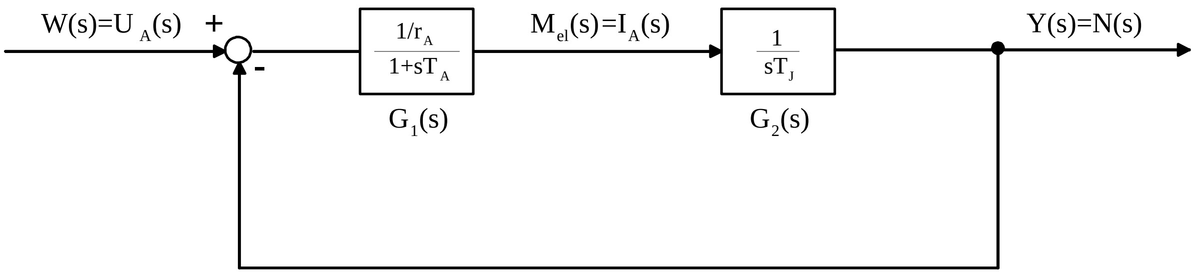


Fig. 15: DC machine, response to set-point changes for M_W=0

$$\frac{Y(s)}{W(s)} \mathrel{\hat{=}} \frac{N(s)}{U_A(s)} \mathrel{\hat{=}} \frac{\frac{1/r_A}{1+sT_A} \cdot \frac{1}{sT_J}}{1+\frac{1/r_A}{1+sT_A} \cdot \frac{1}{sT_J}} = \frac{1}{1+(1+sT_A) \cdot sT_J \cdot r_A} \tag{2.39}$$

with the mechanical time constant:

$$T_m = T_J \cdot r_A = \frac{J \cdot \Omega_0}{M_N} \cdot \frac{R_A}{U_N / I_N} = \frac{J \cdot \Omega_0}{M_N \cdot \frac{U_N}{R_A} \Big/ I_N} = \frac{J \cdot \Omega_0}{M_K} \tag{2.40}$$

follows:

$$\frac{N(s)}{U_A(s)} = \frac{1}{1+sT_m + s^2 T_m T_A} = \frac{1}{T_m T_A \cdot \left(s^2 + \frac{s}{T_A} + \frac{1}{T_A T_m} \right)} \tag{2.41}$$

At the time t = 0 a step change of the reference input variable is applied. For example the stationary machine is connected to rated voltage (coarse start up).

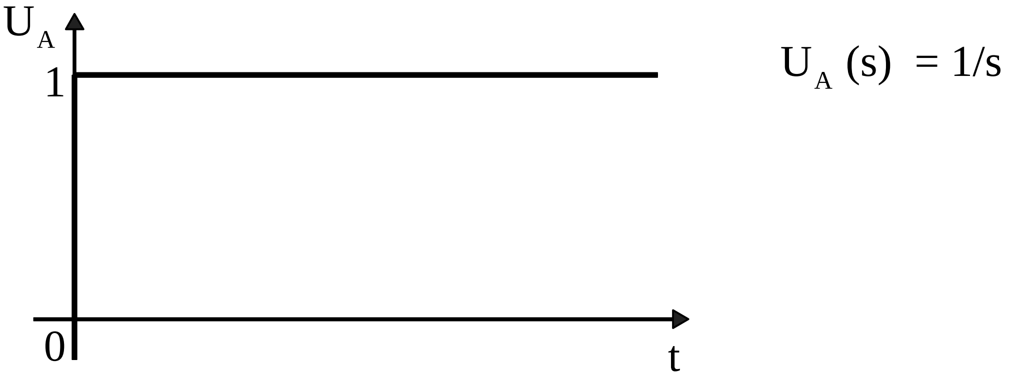


Fig. 16: DC machine, coarse start up

It is to be set:

$$\frac{1}{T_A T_m} = w_0^2 \tag{2.42}$$

$$\frac{1}{T_A} = 2 \cdot D \cdot w_0 = \frac{2 \cdot D}{\sqrt{T_A T_m}} \Rightarrow D = \sqrt{\frac{T_m}{4T_A}} \tag{2.43}$$

Then the speed characteristic is achieved by inverse transformation of

$$n(t) = L^{-1}\left\{\frac{1}{s} \cdot \frac{w_0^2}{s^2 + 2sDw_0 + w_0^2}\right\} \tag{2.44}$$

Regarding the periodic case

$$D = \sqrt{\frac{T_m}{4T_A}} < 1, \tag{2.45}$$

the following solution is derived:

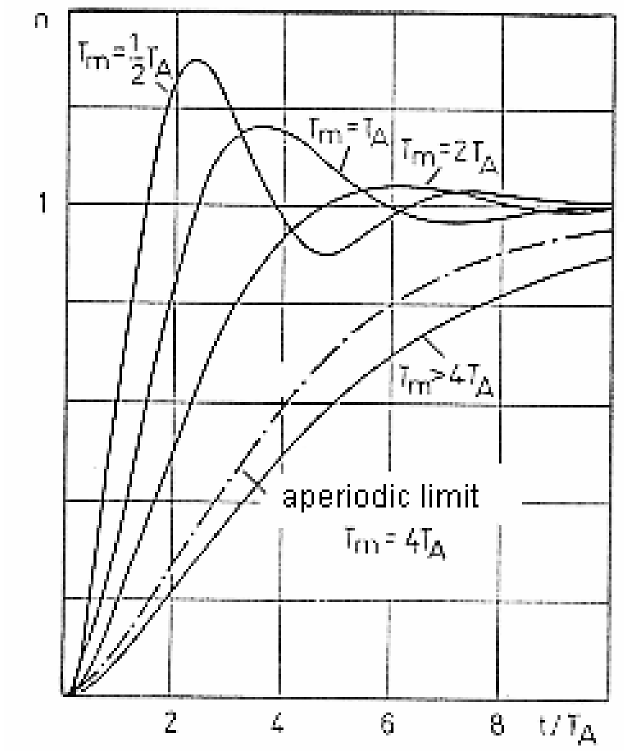
$$n(t) = 1 - \frac{e^{-Dw_0 t}}{\sqrt{1-D^2}} \sin\left(w_0 \sqrt{1-D^2}\, t + \arcsin\sqrt{1-D^2}\right) \tag{2.46}$$

The current is:

$$I_A(s) = sT_J N(s) \tag{2.47}$$

This means a differentiation in the inverse transformation:

$$i_A(t) = L^{-1}\{sT_J N(s)\} = T_J \frac{dn}{dt} = \frac{1}{r_A} T_m \frac{dn}{dt} \tag{2.48}$$

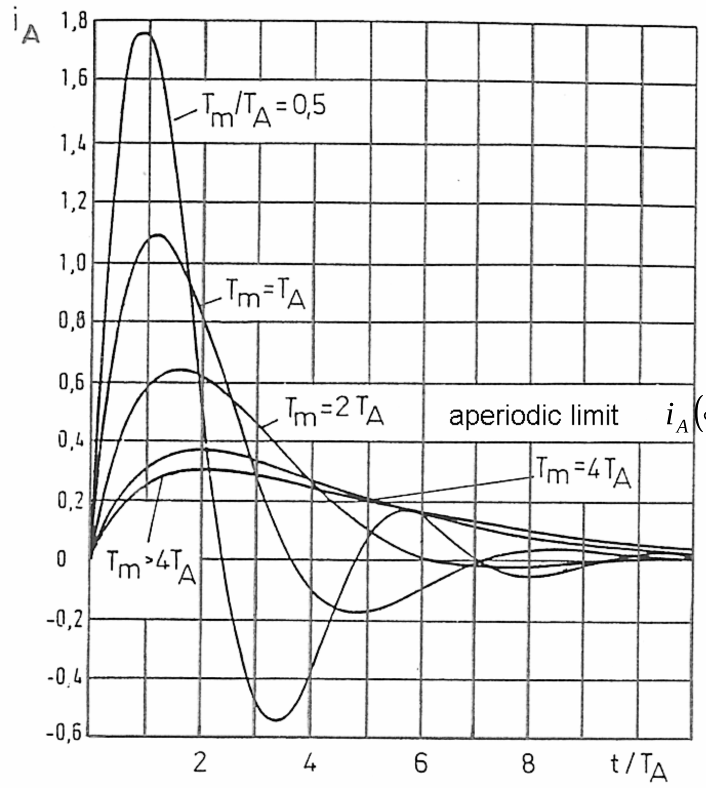


$n(\infty) = 1$

oscillation for $\frac{T_m}{4T_A} < 1$

aperiodic limit for $\frac{T_m}{4T_A} = 1$

Fig. 17: time characteristic of rotational speed for different $\frac{T_m}{T_A}$



$i_A(\infty) = 0$

Fig. 18: time characteristic of the armature current for different $\frac{T_m}{T_A}$

If the machine operates at steady-state no-load condition and the armature voltages is changed, then the current increases, to achieve the new speed. If the speed is achieved, the induced voltage has achieved its new steady-state value and the armature current decreases to zero again. Depending on the magnitude of the damping this process causes oscillation or is aperiodic.

2.) <u>Response to disturbances:</u>

The influence of the disturbance, i.e. the speed variation if the machine is loaded, can be obtained for $W \mathrel{\hat{=}} U_A = 0$:
The physical interpretation is a stationary machine (n = 0), which is loaded with a certain torque. The reaction is a falling-off in speed Δn. By linear superposition any initial condition can be added, i.e. n = 1.

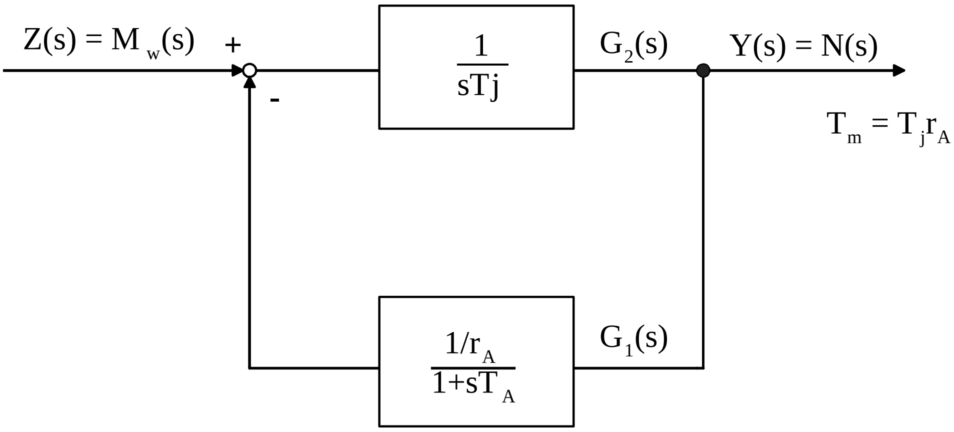


Fig. 19: DC machine, structural diagram due to response to disturbances

$$\frac{Y(s)}{Z(s)} = \frac{\Delta N(s)}{M_W(s)} = \frac{\frac{1}{sT_j}}{1 + \frac{1/r_A}{1+sT_A} \cdot \frac{1}{sT_J}} = \frac{r_A \cdot (1+sT_A)}{1+(1+sT_A)sT_J r_A} = \frac{r_A(1+sT_A)}{1+sT_m+s^2 T_m T_A} \qquad (2.49)$$

For example the machine is loaded with rated torque $M_w(s) = \frac{1}{s}$:

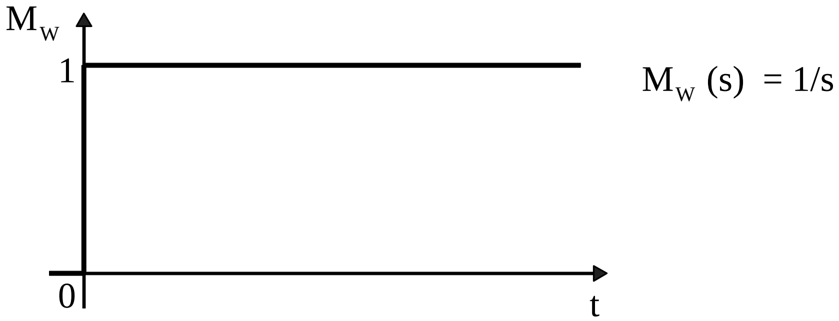


Fig. 20: load with rated torque

The speed variation is:

$$\Delta n = L^{-1}\left\{\frac{r_A(1+sT_A)}{s(1+sT_m+s^2 T_m T_A)}\right\} = r_A L^{-1}\{N(s) + sT_A N(s)\} = r_A\left(n(t) + T_A \frac{dn(t)}{dt}\right) \qquad (2.50)$$

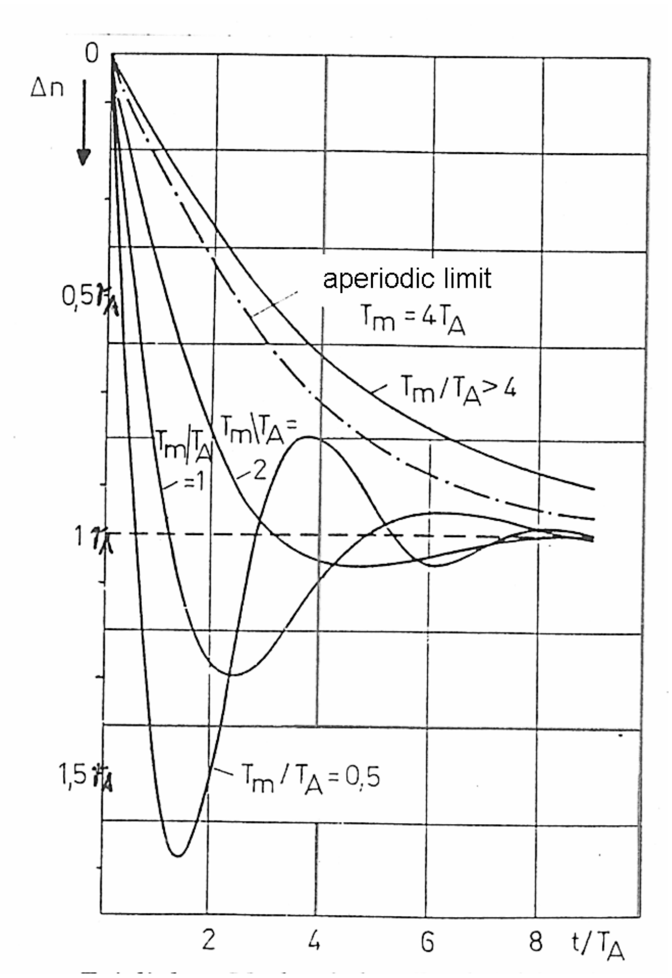


Fig. 21: time characteristic of the speed variation for different $\frac{T_m}{T_A}$

The diagram shows the related falling-off in speed if the machine is loaded with the rated torque. Depending on the magnitude of the damping this process causes oscillation or is aperiodic. In steady-state operation with rated values it is:

$$n = \frac{u_A - r_A \cdot i_A}{i_F} \tag{2.51}$$

$$u_A = 1,\ i_A = 1,\ i_F = 1 \tag{2.52}$$

$$n = 1 - r_A \tag{2.53}$$

$$\Delta n = 1 - n = r_A \tag{2.54}$$

The falling-off in speed if the machine is loaded corresponds to the referred armature resistance r_A.

3.) <u>Results of the analysis:</u>

Separately excited DC machines with energy stores L_A and J are

- oscillating systems for

$$D = \sqrt{\frac{T_m}{4T_A}} < 1 \tag{2.55}$$

- aperiodic characteristics are obtained if the armature voltage or the load changes for

$$D = \sqrt{\frac{T_m}{4T_A}} > 1 \tag{2.56}$$

If a change of state appears to separately excited or permanent excited DC machines, the reaction is a transient phenomenon with an alternately exchange of kinetic energy of the rotor and magnetic energy of the armature winding in the form of periodic oscillations. This can be compared to the reaction of spring-mass system to a change of state. The damping is responsible for the declination of the oscillation. The choice of a suitable current and speed sensor is discussed later on.

2.4 Coarse-step connection of DC shunt machines

If the excitation current changes, the set of differential equations becomes non-linear and can not be solved analytically anymore. Therefore computational algorithms are utilized for analyzing transient phenomena. One phenomenon is the coarse-step connection of DC shunt-wound machines: a stationary machine is connected to the power supply at t = 0.

The time characteristic of speed, armature current, excitation current and torque is intended to be calculated in the following.

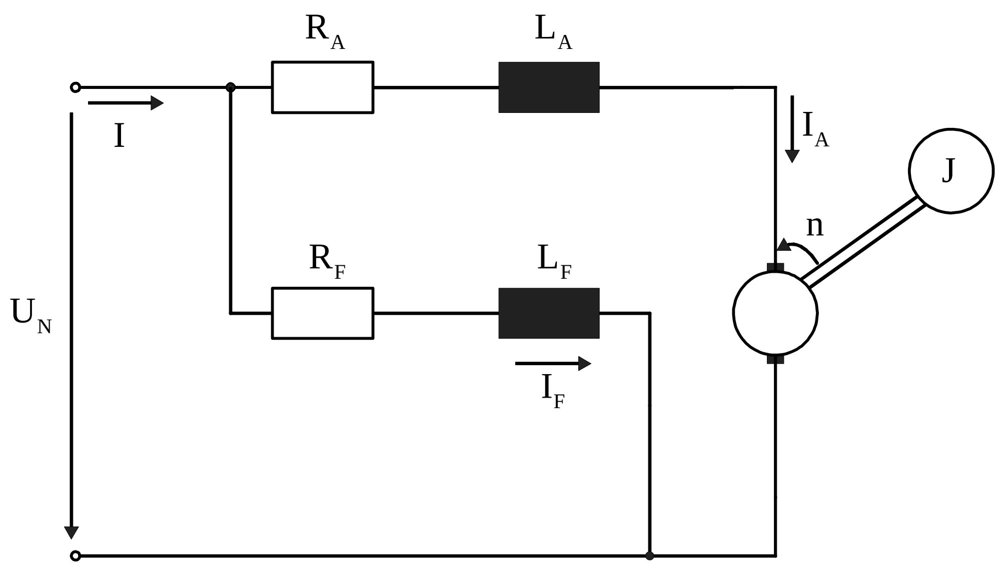


Fig. 22: DC shunt machine, equivalent circuit diagram (ecd)

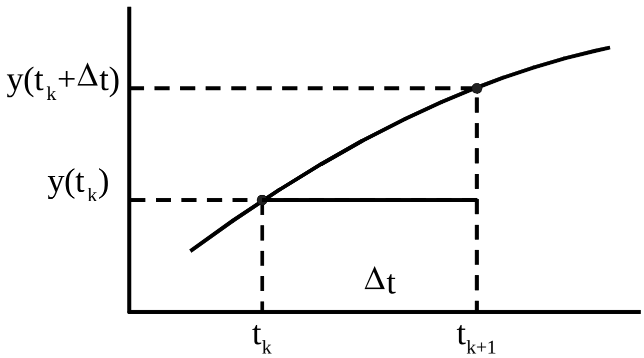
The general based set of equations in state formulation with $u_A = u_F = 1$ is applied. The iron saturation is neglected. The machine should be not loaded $m_W = 0$.

$$T_A \frac{di_A}{dt} = \frac{1}{r_A}(1 - n \cdot i_F) - i_A \qquad (2.57)$$

$$T_F \frac{di_F}{dt} = \frac{1}{r_F} - i_F \qquad (2.58)$$

$$T_J \frac{dn}{dt} = i_A \cdot i_F \qquad (2.59)$$

A stepwise numerical integration has to be performed on the computer.



A simple method is for example the Euler-Cauchy method with a given initial value problem in the form of $y' = f(t, y(t))$ with the initial condition $y(0) = y_0$.

Fig. 23: numerical solution (Euler-Cauchy method)

With

$$\frac{y(t_k + \Delta t) - y(t_k)}{\Delta t} = y'(t_k) \qquad (2.60)$$

follows

$$y(t_{k+1}) = y(t_k) + \Delta t y'(t_k). \qquad (2.61)$$

With that presupposition, a function $y(t)$ at a time $t_{k+1} = t_k + \Delta t$ can be calculated, if the function and its derivation are known at a time t_k.

It is practical to choose the same discretization for each Δt. With $y(t_k) = y(k)$ the following set of equations is obtained:

$$T_A \frac{i_A(k+1) - i_A(k)}{\Delta t} = \frac{1}{r_A}(1 - n(k) \cdot i_F(k)) - i_A(k) \qquad (2.62)$$

$$T_F \frac{i_F(k+1) - i_F(k)}{\Delta t} = \frac{1}{r_F} - i_F(k) \qquad (2.63)$$

$$T_J \frac{n(k+1) - n(k)}{\Delta t} = i_F(k) \cdot i_A(k) \qquad (2.64)$$

A recursive set of equations is formed, by moving all the variables, which are known at the instant t_k, on the right side and then determining the new variables at the instant t_{k+1}. Starting at t = 0 it is integrated stepwise. As a matter of course the scanning intervals has to be much smaller than the least machine time constant $\Delta t << T$.

Applied to the DC shunt-wound machine the following recursive set of equations is achieved:

$$i_A(k+1) = \left(1 - \frac{\Delta t}{T_A}\right) i_A(k) + \frac{\Delta t}{T_A} \cdot \frac{(1 - n(k) \cdot i_F(k))}{r_A} \tag{2.65}$$

$$i_F(k+1) = i_F(k) + \frac{\Delta t}{T_F} \cdot \left(\frac{1}{r_F} - i_F(k)\right) \tag{2.66}$$

$$n(k+1) = n(k) + \frac{\Delta t}{T_J} \cdot i_F(k) \cdot i_A(k) \tag{2.67}$$

The following numerical values were used in the example shown in Fig. 24:

- initial conditions: $i_A(0) = i_F(0) = n(0) = 0$
- scanning time: $\Delta t = 0.1ms$
- time constants: $T_A = 13ms$, $T_F = 0.42s$, $T_J = 0.57s$
- resistances: $r_A = 0.11$, $r_F = 1$

A stepwise calculation results:

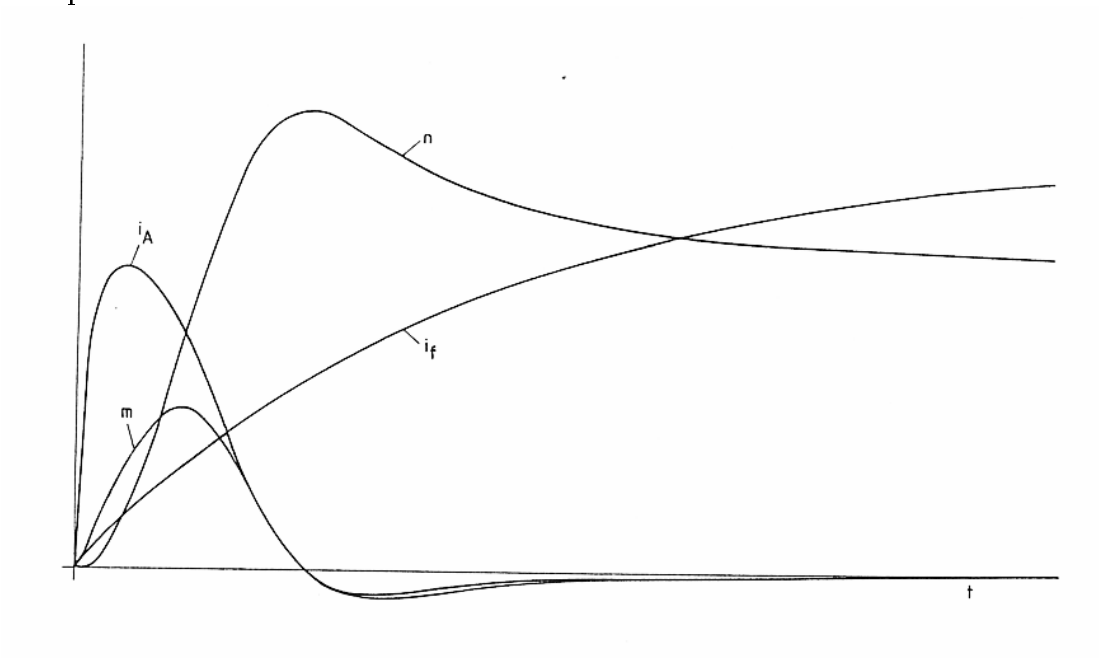


Fig. 24: time characteristics of m, n, i_A, i_f

A sharp increase of the armature current and a slow increase of the excitation current lead to a reduced torque during the acceleration. Even though an aperiodic characteristic is expected, because of $D = \sqrt{T_m / 4T_A} > 1$, the speed and the armature current overshoot, because the field is generated with a delay.

2.5 Cascade control of converter-fed PM DC machines

The block diagram (Fig. 25) shows the common control system of a converter-fed drive system, consisting of a permanent-field (= PM) DC machine, a line-commutated converter and a cascade control. The speed control loop, which for example is realized as a PI-controller, has a lower-level current control loop, which is also realized as a PI-controller.

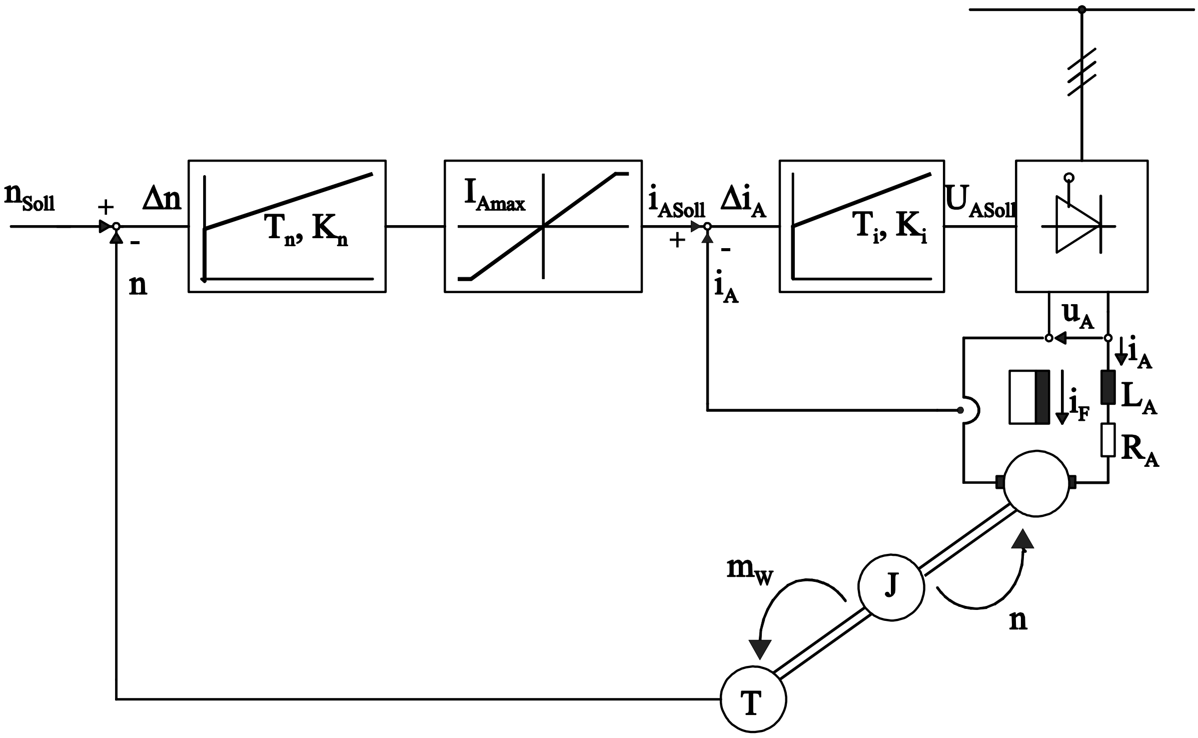


Fig. 25: DC machine, control system diagram

The general set of equations of the permanent-field DC machine in based values with $I_F = I_{FN} = const$, i.e. $i_F = 1$ is:

$$T_A \frac{di_A}{dt} = \frac{1}{r_A}(u_A - n) - i_A \tag{2.68}$$

$$T_J \frac{dn}{dt} = i_A - m_W \tag{2.69}$$

The result of the numeric solution is the following set of equations:

$$i_A(k+1) = \left(1 - \frac{\Delta t}{T_A}\right) \cdot i_A(k) + \frac{\Delta t}{T_A} \cdot \frac{u_A(k) - n(k)}{r_A} \tag{2.70}$$

$$n(k+1) = n(k) + \frac{\Delta t}{T_J} \cdot (i_A(k) - m_W(k)) \tag{2.71}$$

The step-by-step solution of the set of differential equations is done using numeric methods: Then the PI-controllers have to be discretized as follows (illustration due to Fig. 26):

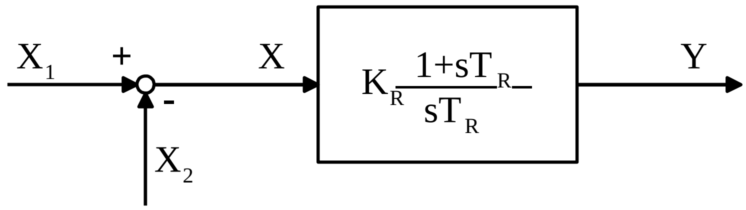


Fig. 26: PI controller

$$\frac{Y(s)}{X(s)} = G(s) = K_R \frac{1+sT_R}{sT_R} \tag{2.72}$$

$$sT_R Y(s) = K_R (1+sT_R) X(s) \tag{2.73}$$

$$T_R \frac{dy}{dt} = K_R \left(x + T_R \frac{dx}{dt} \right) \tag{2.74}$$

$$y(t) = K_R \left(\frac{1}{T_R} \int x(t)dt + x \right) \tag{2.75}$$

$$y(k) = K_R \left(\underbrace{\frac{1}{T_R} \sum_{i=1}^{k-1} x(i)\Delta t(i)}_{I-Anteil} + \underbrace{x(k)}_{P-Anteil} \right) \tag{2.76}$$

The dimensioning of the current and speed controllers is not discussed in detail here. Suitable values for the first attempt are:

- $T_{Ri} \approx T_m = r_A T_J$
- $K_{Ri} \approx 1$
- $T_{Rn} \approx 10 T_{Ri}$
- $K_{Rn} = 5 - 10$

It is important, to switch off the integral-action component if the current is limited. The figure shows the time characteristic of speed and armature current of a permanent-field DC machine during a setpoint step-change of the speed from 0 to 1 and then to -1, i.e. start-up and reversing. The machine has no load. The current is limited to: $\frac{I_A}{I_{AN}} = 2{,}5$.

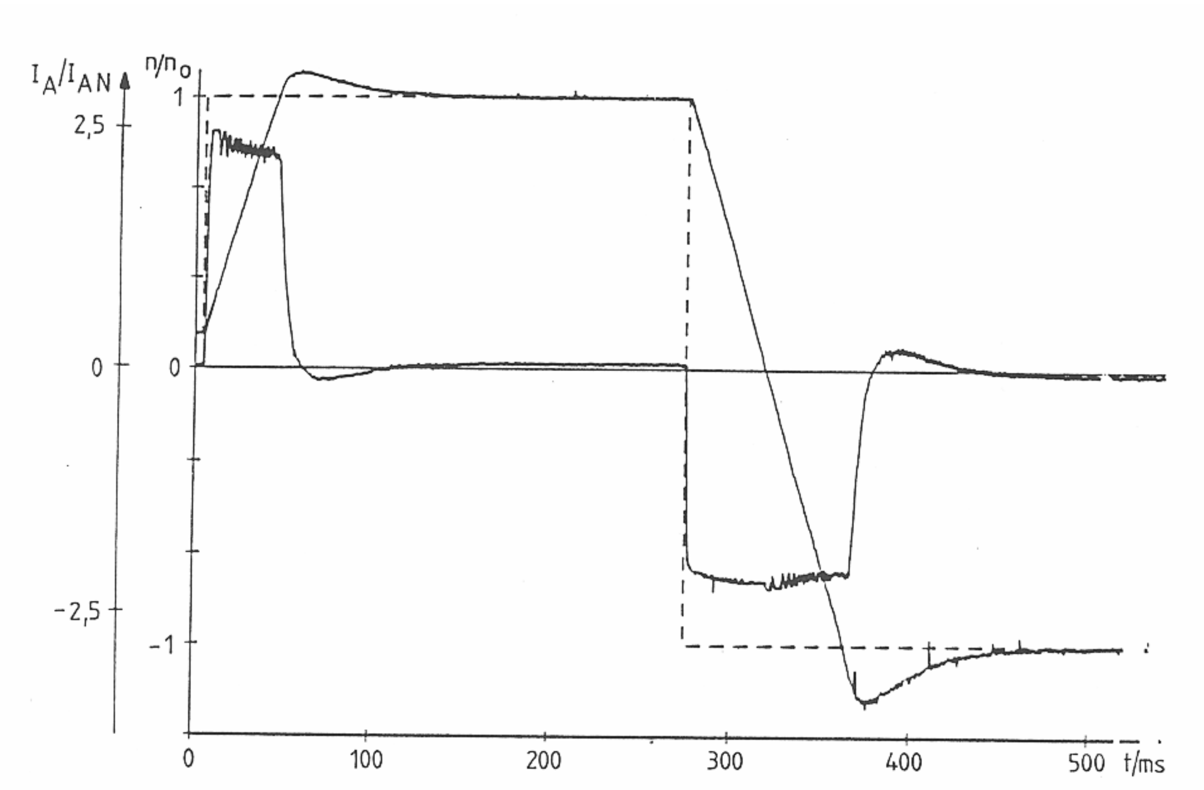


Fig. 27: actual value and desired value (dashed line) of speed and current in based values

Those drives are often used as speed-variable servo drives for machine tools and robots. They are high dynamic, i.e. featuring a slim-line rotor to achieve low moment of inertia $J \sim D^4 \cdot l$ and a small armature time constant, because $T_A = \frac{L_A}{R_A} \sim \frac{1}{d + h_M}$ (because of the permanent-field, the mechanical air-gap d has to be replaced by the magnetic air-gap $d + h_M$). Depending on the type of magnetic material, the ratio $\frac{h_M}{d}$ amounts round about 2 ... 10.
If a position control is applied, the cascade has an additional upper-level control-loop.

2.6 DC series-wound-machine as traction drive in pulse control operation

Speed-variable drives of ground conveyors and electric vehicles with cycle operation are often realized using DC series-wound-machines with DC chopper controllers. Chopper controllers are also used for voltage control of DC-fed train drives.

Fig. 28 shows the general circuit diagram of a clocked DC chopper controller, consisting of a self-commutated thyristor switch or GTO and a freewheeling diode, connected in parallel to the motor.

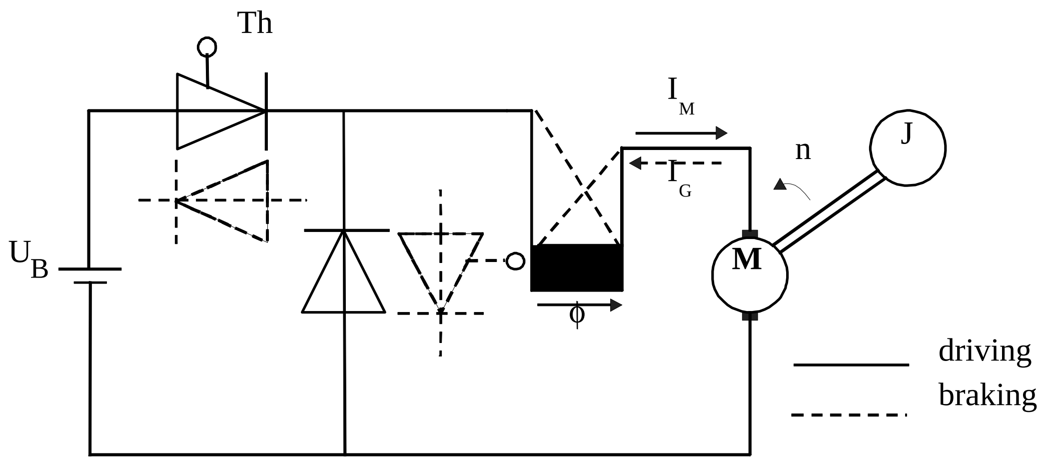


Fig. 28: electric vehicle drive

The method of operation is as follows:

If the thyristor is switched on (conducting), the motor is connected to the battery voltage, the thyristor carries the current and the machine current increases with the time constant $T_{AF} = (L_A + L_F)/(R_A + R_F)$ to the final value $(U - U_i)/(R_A + R_F)$. After opening the thyristor switch, the motor is disconnected from the battery, the diode carries the current and the machine current decreases with the same time constant to zero. Then a new cycle begins.

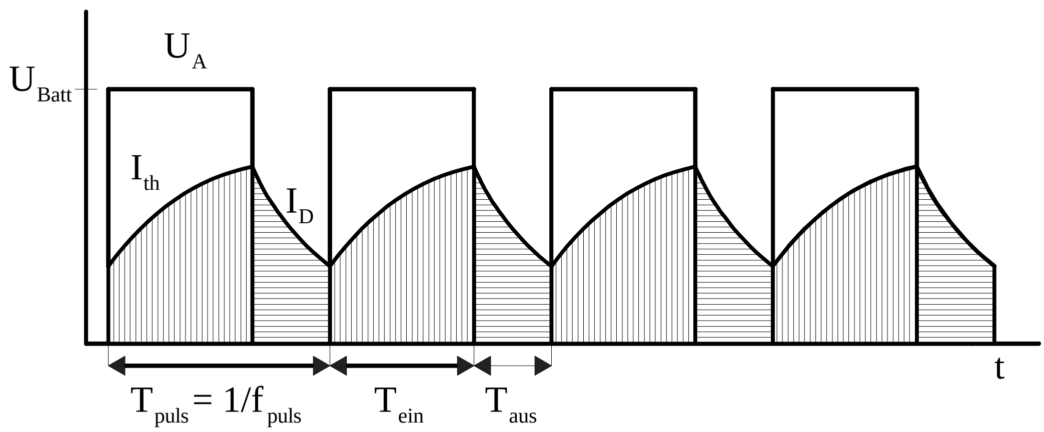


Fig. 29: step-up, step-down converter, duty cycles

Besides the losses in the power semiconductor devices, the chopper controllers operates non-dissipative. With the assumption $T_m >> T_{AF}$, i.e. $n = const$ during a cycle and with negligence of the voltage drop on the windings, it is as a first approximation:

$$n \sim U_A = \frac{1}{T_{Puls}} \int_0^{T_{Puls}} u(t)dt = \frac{1}{T_{Puls}} U_{Batt} T_{ein} \tag{2.77}$$

The mean value of the motor voltage and so at least the speed is controlled by the ratio of conducting time (T_{ein} = T_{on}) and dead time (T_{aus} = T_{off}) of the thyristor. The pulse frequencies are some 100 Hz up to some kHz. The chopper controller allows only one-quadrant operation. For braking, the thyristor and the freewheeling diode has to be interchanged in the circuit using contactors.

3 Induction machine

3.1 Dynamic equation set

To calculate the dynamic behavior of induction machines, the general system of equations for rotating field machines can be utilized. Because of the constant air gap the choice of the angle $a(t)$ is arbitrary.

An arbitrary coordinate system is to be used, whose rotational speed and initial value can be chosen freely

$$a(t) = w_K \cdot t + a_0 , \qquad \frac{da}{dt} = w_K . \tag{3.1 a, b}$$

The mechanical speed ensues to:

$$\frac{dg}{dt} = w = p \cdot \Omega = p \cdot 2 \cdot p \cdot n . \tag{3.2}$$

After converting the rotor quantities on the number of the stator windings, the voltage equations of the induction machine are:

$$\begin{bmatrix} u_{d1} \\ u_{q1} \end{bmatrix} = R_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} + w_K \cdot \begin{bmatrix} -\Psi_{q1} \\ +\Psi_{d1} \end{bmatrix} \tag{3.3}$$

$$\begin{bmatrix} u'_{d2} \\ u'_{q2} \end{bmatrix} = R'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} + (w - w_K) \cdot \begin{bmatrix} +\Psi'_{q2} \\ -\Psi'_{d2} \end{bmatrix} \tag{3.4}$$

flux linkages:

$$\begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \end{bmatrix} = L_1 \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} + L_h \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} \tag{3.5}$$

$$\begin{bmatrix} \Psi'_{d2} \\ \Psi'_{q2} \end{bmatrix} = L'_2 \cdot \begin{bmatrix} i'_{d2} \\ i'_{q2} \end{bmatrix} + L_h \cdot \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} \tag{3.6}$$

torque equations:

$$M_{el} = p \cdot L_h \cdot \left(i_{q1} \cdot i'_{d2} - i_{d1} \cdot i'_{q2} \right) = M_W + \frac{J}{p} \cdot \frac{dw}{dt} \tag{3.7}$$

$$= p \cdot \left(\Psi_{d1} \cdot i_{q1} - \Psi_{q1} \cdot i_{d1} \right) = p \cdot \left(\Psi'_{q2} \cdot i'_{d2} - \Psi'_{d2} \cdot i'_{q2} \right) \tag{3.8}$$

At first a suitable coordinate system is to be chosen, with assumed synchronous rotation with the rotating field in the stator:

$$\frac{da}{dt} = w_K = w_1 \tag{3.9}$$

$$a(t) = w_1 \cdot t + a_0 \,. \tag{3.10}$$

For the inverse transformation into the complex notation, the choice of the constant of integration α_0 is still arbitrary. A reasonable choice appears due to Fig. 30:

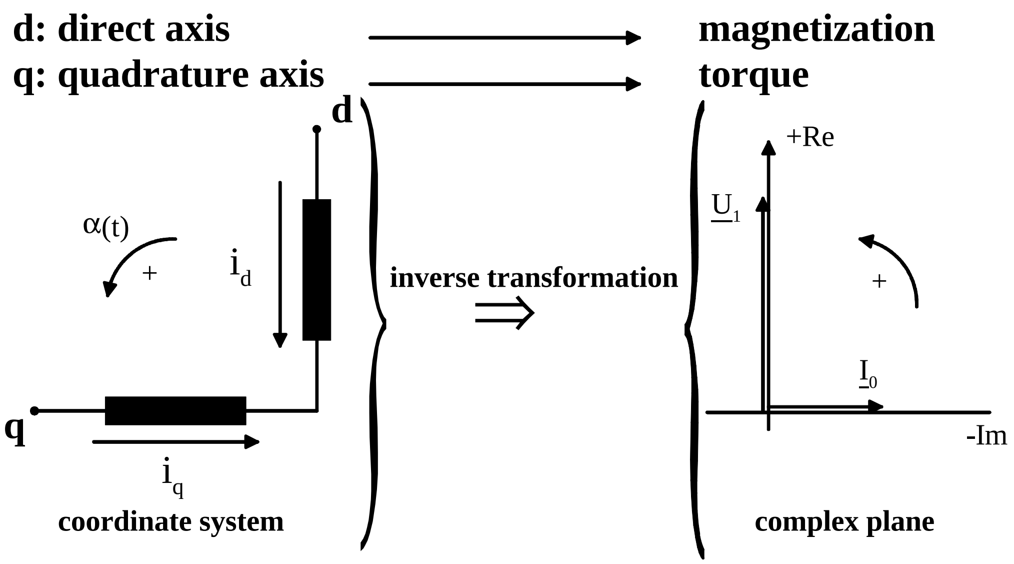


Fig. 30: coordinate transformation

$$u_{u1} = \sqrt{2} \cdot U_1 \cdot \cos(w_1 \cdot t) = \sqrt{\frac{2}{3}} \cdot \left[u_{d1} \cdot \cos(w_1 \cdot t + a_0) - u_{q1} \cdot \sin(w_1 \cdot t + a_0)\right] \tag{3.11}$$

$$\sqrt{2} \cdot U_1 \cdot \mathrm{Re}\left\{e^{jw_1 \cdot t}\right\} = \sqrt{\frac{2}{3}} \cdot \left[u_{d1} \cdot \mathrm{Re}\left\{e^{jw_1 \cdot t} \cdot e^{ja_0}\right\} - u_{q1} \cdot \underbrace{\mathrm{Im}\left\{e^{jw_1 \cdot t} \cdot e^{ja_0}\right\}}_{\cdot \mathrm{Re}\left\{(-j) \cdot e^{jw_1 \cdot t} \cdot e^{ja_0}\right\}}\right] \tag{3.12}$$

$$\underline{U}_1 = \frac{u_{d1}}{\sqrt{3}} \cdot e^{ja_0} + j \cdot \frac{u_{q1}}{\sqrt{3}} \cdot e^{ja_0} \tag{3.13}$$

Definition for induction machines: $a_0 = -\frac{p}{2}$, $\quad e^{ja_0} = e^{-j\frac{p}{2}} = -j$. Then follows:

- quadrature axis (q-axis), in which the torque is generated, corresponds to the real axis.
- direct axis (d-axis), in which the machine is magnetized, corresponds to the imaginary axis.

$$\underline{U}_1 = \frac{u_{q1}}{\sqrt{3}} - j \cdot \frac{u_{d1}}{\sqrt{3}} = \underline{U}_{q1} + \underline{U}_{d1} \tag{3.14}$$

Currents are defined in the same way:

$$\underline{I}_1 = \frac{i_{q1}}{\sqrt{3}} - j \cdot \frac{i_{d1}}{\sqrt{3}} = \underline{I}_{q1} + \underline{I}_{d1} \tag{3.15}$$

The rotor is considered in the same manner:

$$\underline{U}'_2 = \frac{u'_{q2}}{\sqrt{3}} - j \cdot \frac{u'_{d2}}{\sqrt{3}} = \underline{U}'_{q2} + \underline{U}'_{d2} \tag{3.16}$$

$$\underline{I}'_2 = \frac{i'_{q2}}{\sqrt{3}} - j \cdot \frac{i'_{d2}}{\sqrt{3}} = \underline{I}'_{q2} + \underline{I}'_{d2} \tag{3.17}$$

As the currents i_{q1} and i_{d1} in the rotating two-phase system are direct currents in the steady-state operation, the currents $\underline{I}_{q1}$ and $\underline{I}_{d1}$ in the complex notation are currents with system frequency, whose r.m.s. value equals $\frac{1}{\sqrt{3}}$ times the DC value and whose phase angle is determined by $a_0 = -\frac{p}{2}$.

- $\underline{I}_{q1}$ and $\underline{I}'_{q2}$ are active currents in the complex notation.
- $\underline{I}_{d1}$ and $\underline{I}'_{d2}$ are reactive currents in the complex notation.

3.2 Steady state operation

In steady-state operation the flux linkages in the rotating system are constant, i.e.

$$\frac{d\Psi}{dt} = 0 \tag{3.18}$$

and the speed is constant, i.e.

$$\frac{dw}{dt} = 0 \tag{3.19}$$

Torque equation and voltage equation are therefore decoupled and independent

$$u_{d1} = R_1 \cdot i_{d1} - w_1 \cdot \Psi_{q1} \qquad u_{id} = -w_1 \cdot \Psi_{q1} \tag{3.20}$$

$$u_{q1} = R_1 \cdot i_{q1} + w_1 \cdot \Psi_{d1} \qquad u_{iq} = w_1 \cdot \Psi_{d1} \tag{3.21}$$

$$u'_{d2} = R'_2 \cdot i'_{d2} + (w - w_1) \cdot \Psi'_{q2} \tag{3.22}$$

$$u'_{q2} = R'_2 \cdot i'_{q2} - (w - w_1) \cdot \Psi'_{d2} \tag{3.23}$$

$$M_{el} = p \cdot (\Psi_{d1} \cdot i_{q1} - i_{d1} \cdot \Psi_{q1}) \tag{3.24}$$

The inverse transformation of the voltage equations provides:

$$\underline{U}_1 = \frac{R_1 \cdot i_{q1} + w_1 \cdot \left(L_1 \cdot i_{d1} + L_h \cdot i'_{d2}\right)}{\sqrt{3}} - j\frac{R_1 \cdot i_{d1} - w_1 \cdot \left(L_1 \cdot i_{q1} + L_h \cdot i'_{q2}\right)}{\sqrt{3}} \tag{3.25}$$

$$\underline{U}'_2 = \frac{R'_2 \cdot i'_{q2} + \frac{w_1 - w}{w_1} w_1 \cdot \left(L'_2 \cdot i'_{d2} + L_h \cdot i_{d1}\right)}{\sqrt{3}} - j\frac{R'_2 \cdot i'_{d2} - \frac{w_1 - w}{w_1} w_1 \cdot \left(L'_2 \cdot i'_{q2} + L_h \cdot i_{q1}\right)}{\sqrt{3}} \tag{3.26}$$

$$\underline{U}_1 = R_1 \cdot \underline{I}_{q1} + j \cdot X_1 \cdot \underline{I}_{d1} + j \cdot X_h \cdot \underline{I}'_{d2} + R_1 \cdot \underline{I}_{d1} + j \cdot X_1 \cdot \underline{I}_{q1} + j \cdot X_h \cdot \underline{I}'_{q2} \tag{3.27}$$

$$\underline{U}'_2 = R'_2 \cdot \underline{I}'_{q2} + j \cdot s \cdot X'_2 \cdot \underline{I}'_{d2} + j \cdot s \cdot X_h \cdot \underline{I}_{d1} + R'_2 \cdot \underline{I}'_{d2} + j \cdot s \cdot X'_2 \cdot \underline{I}'_{q2} + j \cdot s \cdot X_h \cdot \underline{I}_{q1} \tag{3.28}$$

$$\underline{U}_1 = R_1 \cdot \underline{I}_1 + j \cdot X_1 \cdot \underline{I}_1 + j \cdot X_h \cdot \underline{I}'_2 \tag{3.29}$$

$$\underline{U}'_2 = R'_2 \cdot \underline{I}'_2 + j \cdot s \cdot X'_2 \cdot \underline{I}'_2 + j \cdot s \cdot X_h \cdot \underline{I}_1 \tag{3.30}$$

Finally the well-known voltage equations of the induction machine is achieved, corresponding to the symmetric equivalent circuit diagram (ecd) as depicted in Fig. 31.

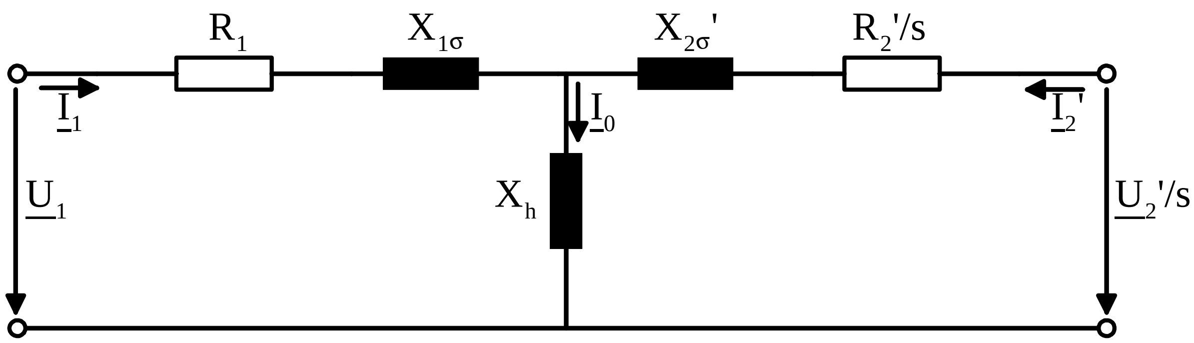


Fig. 31: induction machine, equivalent circuit diagram (ecd)

Torque ensues to:

$$M_{el} = \frac{3p}{w_1} \cdot \left(\frac{w_1 \cdot \Psi_{d1}}{\sqrt{3}} \cdot \frac{i_{q1}}{\sqrt{3}} - \frac{w_1 \cdot \Psi_{q1}}{\sqrt{3}} \cdot \frac{i_{d1}}{\sqrt{3}} \right) = \frac{3p}{w_1} \cdot \left(U_{iq} \cdot I_q + U_{id} \cdot I_d\right) \tag{3.31}$$

3.3 Rapid acceleration, sudden load change

Using numerical integration methods on the computer, now the rapid acceleration of an induction machine with squirrel-cage rotor is calculated. At the time $t = 0$ the machine in standstill is connected to the supply voltage. It is assumed, that the supplying system is a stiff system and the machine is loaded only with its moment of inertia. Afterwards a sudden change load change with rated torque occurs.

The parameter $a(t)$ is chosen as $a(t) = w_1 \cdot t - \frac{p}{2}$ and the system of equation is to be transformed in state form. For the squirrel-cage rotor is considered: $u_{d2}^{'} = u_{q2}^{'} = 0$

$$\frac{d\Psi_{d1}}{dt} = u_{d1} - i_{d1} \cdot R_1 + w_1 \cdot \Psi_{q1} \tag{3.32}$$

$$\frac{d\Psi_{q1}}{dt} = u_{q1} - i_{q1} \cdot R_1 - w_1 \cdot \Psi_{d1} \tag{3.33}$$

$$\frac{d\Psi_{d2}^{'}}{dt} = -i_{d2}^{'} \cdot R_2^{'} - (w - w_1) \cdot \Psi_{q2}^{'} \tag{3.34}$$

$$\frac{d\Psi_{q2}^{'}}{dt} = -i_{q2}^{'} \cdot R_2^{'} + (w - w_1) \cdot \Psi_{d2}^{'} \tag{3.35}$$

$$\frac{dw}{dt} = \frac{p}{J} \cdot \left[p \cdot \left(\Psi_{d1} \cdot i_{q1} - \Psi_{q1} \cdot i_{d1} \right) - M_W \right] \tag{3.36}$$

Currents result from the inverse inductance matrix.

$$\begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2}^{'} \\ i_{q2}^{'} \end{bmatrix} = \underbrace{\frac{1-s}{s \cdot L_{1h}} \cdot \begin{bmatrix} (1+s_2) & 0 & -1 & 0 \\ 0 & (1+s_2) & 0 & -1 \\ -1 & 0 & (1+s_1) & 0 \\ 0 & -1 & 0 & (1+s_1) \end{bmatrix}}_{[L]^{-1}} \cdot \begin{bmatrix} \Psi_{d1} \\ \Psi_{q1} \\ \Psi_{d2}^{'} \\ \Psi_{q2}^{'} \end{bmatrix} \tag{3.37}$$

The following structure diagram for induction machines, shown in Fig. 32, accords to the discussed behavior:

Values are defined as:

- excitation values: u_{d1}, u_{q1}, M_W
- state values: Ψ_{d1}, Ψ_{q1}, $\Psi_{d2}^{'}$, $\Psi_{q2}^{'}$, w
- default: $w_K = w_1$

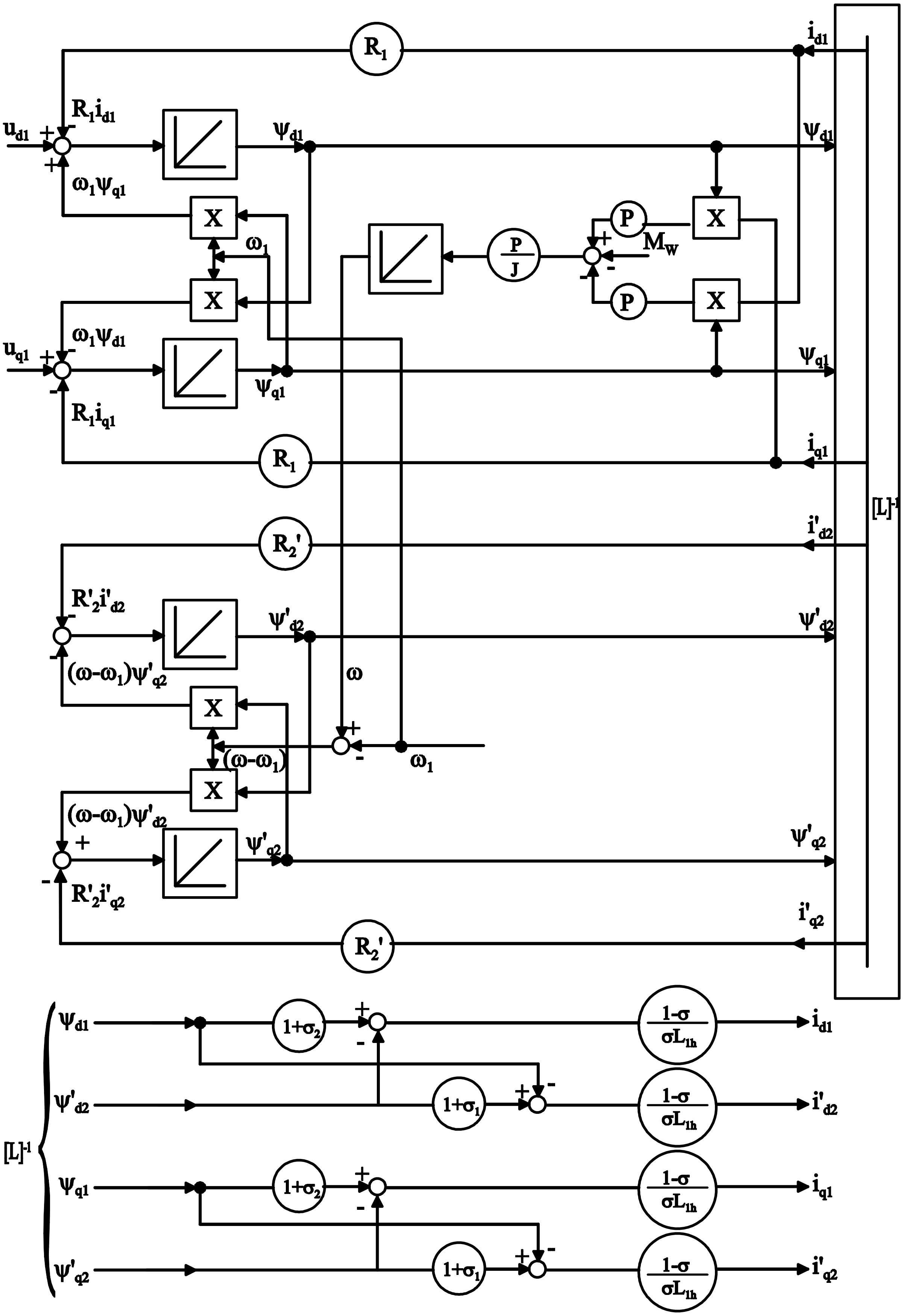


Fig. 32: induction machine, structural diagram

Initial conditions for $t < 0$ in case of machine being turned-off appear as:

$$i_{d1} = i_{q1} = i'_{d2} = i'_{q2} = w = 0 \tag{3.38}$$

$$u_{d1} = u_{q1} = M_W = 0 \tag{3.39}$$

Excitation values of the rapid acceleration case of induction machines are (for $t > 0$):

$$u_{d1} = 0\,, \qquad u_{q1} = \sqrt{3}\cdot U_1\,, \qquad M_W = 0 \qquad \text{(3.40 a-c)}$$

After rapidly starting up, the induction machine is operating at no-load and the initial conditions before the sudden load change are considered:

$$i_{d1} = \sqrt{3}\cdot I_0\,, \qquad w = w_1 \qquad \text{(3.41 a,b)}$$

$$i_{q1} = i'_{d2} = i'_{q2} = 0 \tag{3.42}$$

For $t > t_L$ the considered induction machine is to be suddenly loaded with rated torque. The excitation values ensue to:

$$u_{d1} = 0\,, \qquad u_{q1} = \sqrt{3}\cdot U_1\,, \qquad M_W = M_N \qquad \text{(3.43 a-c)}$$

The simulation is based on the following machine data:

- $P_N = 400\,\text{kW}$
- $n_0 = 1000\,\text{min}^{-1}$
- $U_{1N} = 380V$ (verk.)
- $I_0 = 160\,\text{A}$
- $I_{1N} = 715\,\text{A}$
- $s = 0{,}05$
- $\cos j_N = 0{,}90$
- $M_N = 3906\,\text{Nm}$
- $T_m = \dfrac{J\cdot\Omega_0}{M_{kipp}} = 0{,}33s$

In both cases a dynamic transient reaction takes place.

- When it is switched on, the induction machine generates heavy oscillating torques at standstill because of the DC components in combination with high symmetrical short-circuit currents. After the declination the machine runs up (more or less fast, depending on the coupled masses) and adjusts itself with overshoots at no load.
- The sudden load change is braking the machine at first until the electrical torque is built up. Afterwards the machine adjusts itself to a steady state.

The according diagrams (Fig. 33-37) show:

- stator current $i_u(t)$, rotor speed $n(t)$ and electrical torque $M_{el} = f(t)$ as time dependent functions
- dynamic torque-speed characteristic $M_{el} = f(n)$ and the dynamic circle diagram $I_{Wirk} = f(I_{Blind})$

The deviations from the steady-state characteristics are notably.

For that case the induction machine is not practical as a dynamic actuator in a drive system. In the following it will be examined, if it is possible, to let the induction machine have the same dynamic performance as the DC machine.

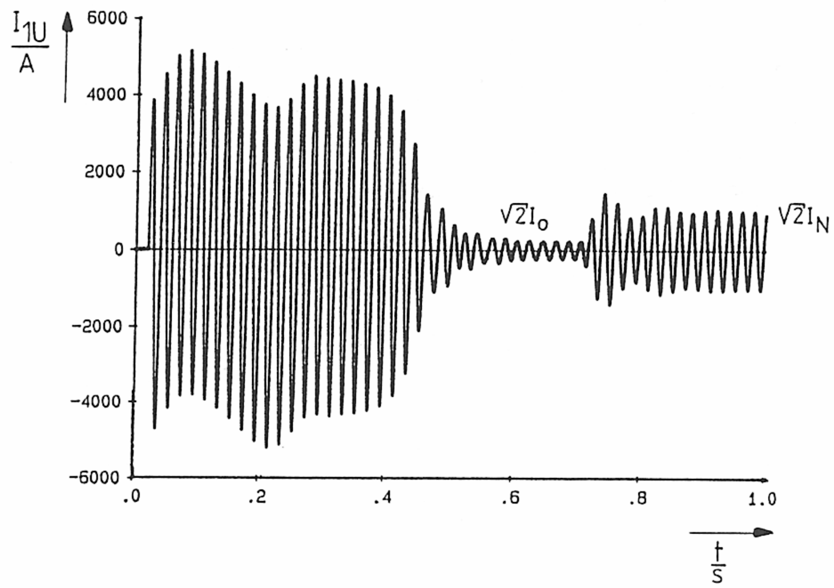


Fig. 33: induction machine, stator current $i_{1u}(t)$ vs. time

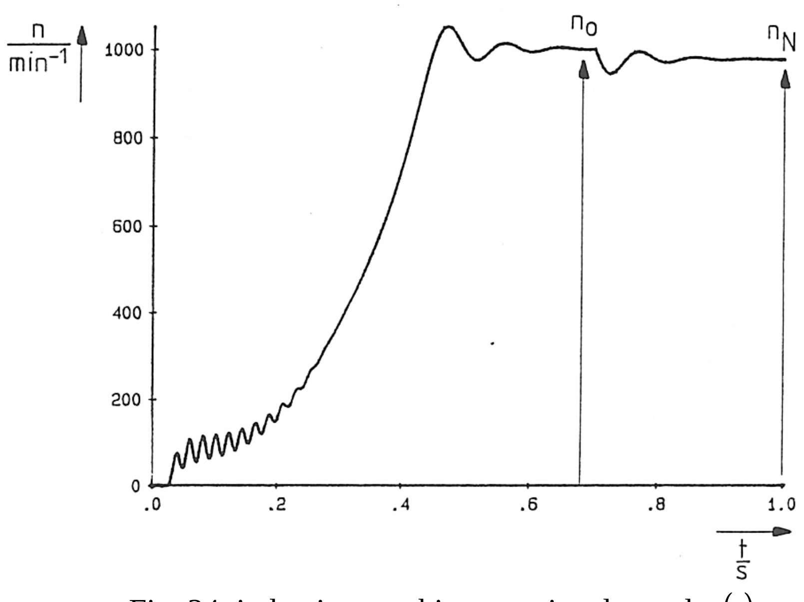


Fig. 34: induction machine, rotational speed $n(t)$ vs. time

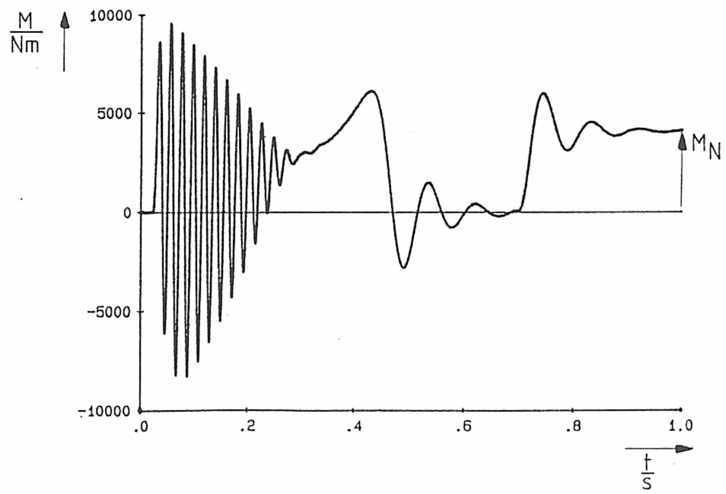


Fig. 35: induction machine, torque $M(t)$ vs. Time

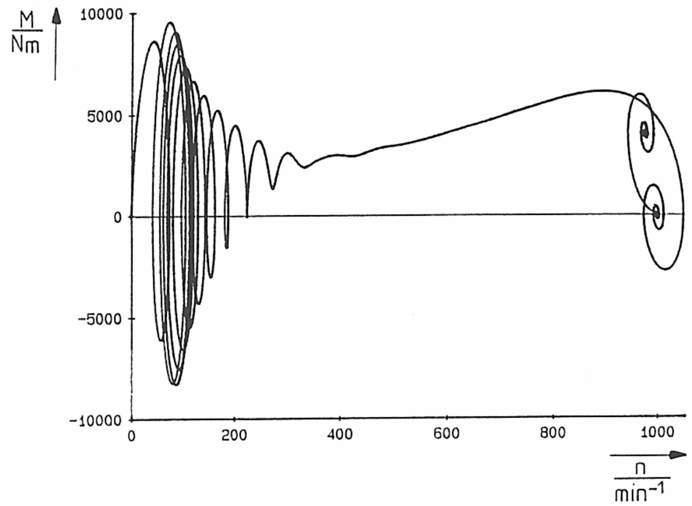


Fig. 36 : induction machine, dynamic speed-torque characteristic $M(n)$

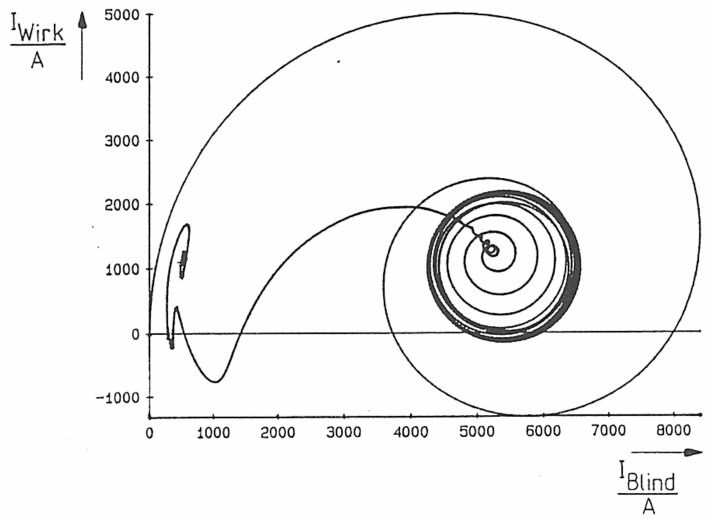


Fig. 37: induction machine, dynamic circle diagram $I_{Wirk}(I_{Blind})$

3.4 Induction machine in field-oriented coordinate system

Because of the effect of the collector, the excitation flux is always perpendicular to the armature current-linkage and their spatial position is stationary.

If the machine has a commutating-field winding and a compensating-field winding, then the armature quadrature-axis field is completely compensated $\Theta_A + \Theta_W + \Theta_K = 0$. Thus the armature flux-linkage in the armature quadrature-axis equals zero $\Psi_{qA} = 0$ and the excitation flux is not influenced by the armature current. Therefore the armature flux-linkage in the direct axis depends only on the field excitation current $\Psi_{dA} \sim I_F$ and the torque is $M_{el} \sim I_A \cdot I_F$.

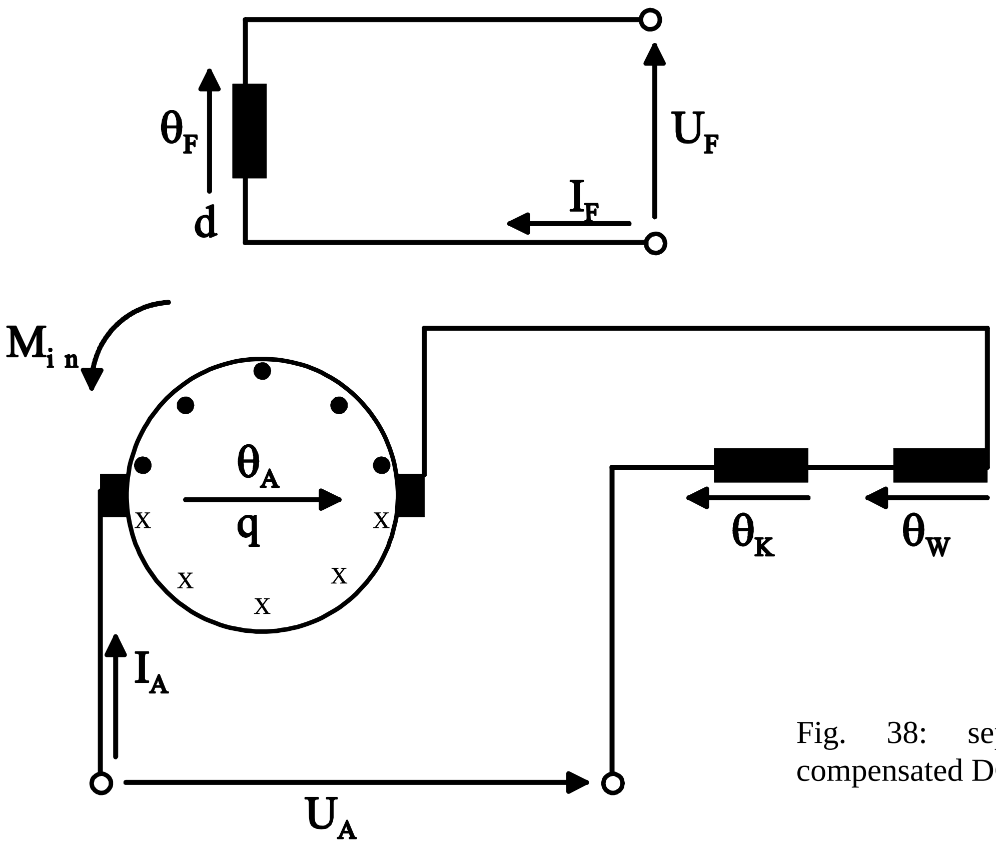


Fig. 38: separately excited, compensated DC machine (VZS)

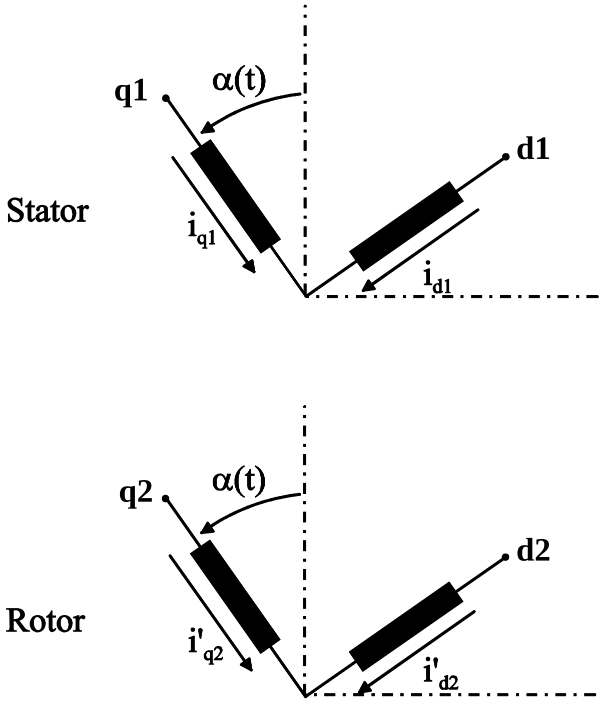
Advantage can be taken of this for the induction machine by choosing a rotor-flux-oriented coordinate system, which is rotating with the speed of the rotor flux

$$a(t) = w_m \cdot t + a_0 \tag{3.44}$$

whereas the instantaneous value of the angular speed

$$w_m = w + w_R \tag{3.45}$$

does not necessarily need to correspond with the stationary value w_1 of the stator field at rated frequency.



A decomposition of the stator and rotor current-linkages direct (i_{d1}, i'_{d2}) and quadrature (i_{q1}, i'_{q2}) components regarding the rotor flux leads to a clear decoupling and permits a suitable control, to inject currents in such a manner, that the rotor flux-linkage in the quadrature axis becomes zero $\Psi'_{q2} = 0$ and the that the rotor flux-linkage in the direct axis only depends on the magnetizing current $\Psi'_{d2} \sim i_m$.

Then the torque is only generated by the perpendicular components of rotor flux and stator active current $M_{el} \sim \Psi'_{d2} \cdot i_{q1}$.

Fig. 39: coordinate system, revolving at rotor flux rotational speed (d=direct, q=quadrature)

A direction convention of the axis (direct/quadrature) is used due to:

- d: direction of the rotor flux
- q: perpendicular to the rotor flux

This is called field-oriented operation. An observer, rotating with the system with α(t), detects the same field distribution and the same torque generation as in comparable DC machines.

The result are simple relations for the controlled variables rotor flux and stator active current. Both variables can be adjusted independently ($\Rightarrow$ compare to: DC machine).

For the rotor flux-linkages it is postulated:

$$\Psi'_{d2} = (1 + s_2) \cdot L_h \cdot i'_{d2} + L_h \cdot i_{d1} \overset{!}{=} L_h \cdot i_m \qquad \text{(direct reference axis)} \tag{3.46}$$

$$\Psi'_{q2} = (1 + s_2) \cdot L_h \cdot i'_{q2} + L_h \cdot i_{q1} \overset{!}{=} 0 \qquad \text{(quadrature axis)} \tag{3.47}$$

Thereby the variable i_m is a user-defined magnetizing current, which is proportional to the rotor flux linkage.

From this follows for the rotor currents:

$$i'_{d2} = \frac{1}{1 + s_2} \cdot (i_m - i_{d1}) \tag{3.48}$$

$$i'_{q2} = \frac{1}{1 + s_2} \cdot (-i_{q1}) \tag{3.49}$$

The angular speeds ensues to:

$$\frac{dg}{dt} = w = p \cdot \Omega \tag{3.50}$$

$$\frac{da}{dt} = w_K = w_m = w + w_R \tag{3.51}$$

whereas w_R is the angular frequency of the rotor currents.

The rotor voltage equations are:

$$0 = R_2^{'} \cdot i_{d2}^{'} + \frac{d\Psi_{d2}^{'}}{dt} + \frac{d(g-a)}{dt} \cdot \Psi_{q2}^{'} \tag{3.52}$$

$$0 = R_2^{'} \cdot i_{q2}^{'} + \frac{d\Psi_{q2}^{'}}{dt} - \frac{d(g-a)}{dt} \cdot \Psi_{d2}^{'} \tag{3.53}$$

After pasting the above relations follows:

$$0 = R_2^{'} \cdot \frac{1}{1+s_2} \cdot (i_m - i_{d1}) + L_h \cdot \frac{di_m}{dt} \tag{3.54}$$

$$0 = R_2^{'} \cdot \frac{1}{1+s_2} \cdot (-i_{q1}) + w_R \cdot L_h \cdot i_m \tag{3.55}$$

With the rotor time constant T_2 due to

$$T_2 = \frac{(1+s_2) \cdot L_h}{R_2^{'}} = \frac{L_2^{'}}{R_2^{'}}, \tag{3.56}$$

the rotor equations in field-oriented coordinates are obtained:

i_{d1} controls i_m : $$T_2 \cdot \frac{di_m}{dt} + i_m = i_{d1} \tag{3.57}$$

w_R is proportional to i_{q1} : $$w_R = \frac{i_{q1}}{T_2 \cdot i_m} = w_m - w \tag{3.58}$$

The torque equation in field-oriented coordinates is:

$$M_{el} = p \cdot \left(\Psi_{q2}^{'} \cdot i_{d2}^{'} - \Psi_{d2}^{'} \cdot i_{q2}^{'}\right) = p \cdot \frac{\Psi_{d2}^{'}}{1+s_2} \cdot i_{q1} \tag{3.59}$$

M_{el} is proportional to i_{q1} and i_m :

$$M_{el} = \frac{p \cdot L_h}{1+s_2} \cdot i_m \cdot i_{q1} = M_W + \frac{J}{p} \cdot \frac{dw}{dt} \tag{3.60}$$

The stator voltage equations do not have to be regarded further on, if the stator currents are injected by power converters with high switching frequencies and short sampling times (servo-converter).

The equation for i_m shows, that the direct component of the stator current controls the rotor flux. The large rotor time constant is the controlling time constant ($\Rightarrow$ compare with the field winding of DC machines). Therefore the magnitude of the rotor flux is not suitable for rapid control processes.
The equation for w_R shows, how the angular speed of the rotor flux is composed of the mechanical angular speed of the rotor and the angular speed of the slip, which results from the active current component of the stator current and the magnitude of the rotor flux.

The torque equation describes the mechanical dynamic response and the torque generation, which now results from the direct flux and the quadrature current ($\Rightarrow$ compare with DC machines). If $\Psi'_{d2} = const$ applies, torque M_{el} und the angular speed w_R are proportional to i_{q1}. These three equations describe the model of the induction machine in field-oriented coordinates.

In analogy to DC machines, an equivalent structure diagram for induction machines in field-oriented coordinates with injected stator currents can be found. This means, that under certain conditions induction machines behave like separately excited DC machines with vanishing time constant. At constant rotor flux the torque generation follows the quadrature current instantaneously and the rotor flux can be adjusted solely with the direct current component.

$$w_m \cdot T_2 \cdot i_m - w \cdot T_2 \cdot i_m = i_{q1} \tag{3.61}$$

$$T_2 \cdot \frac{di_m}{dt} + i_m = i_{d1} \tag{3.62}$$

$$M_{el} = \frac{p \cdot L_h}{1 + s_2} \cdot i_m \cdot i_{q1} = M_W + \frac{J}{p} \cdot \frac{dw}{dt} \tag{3.63}$$

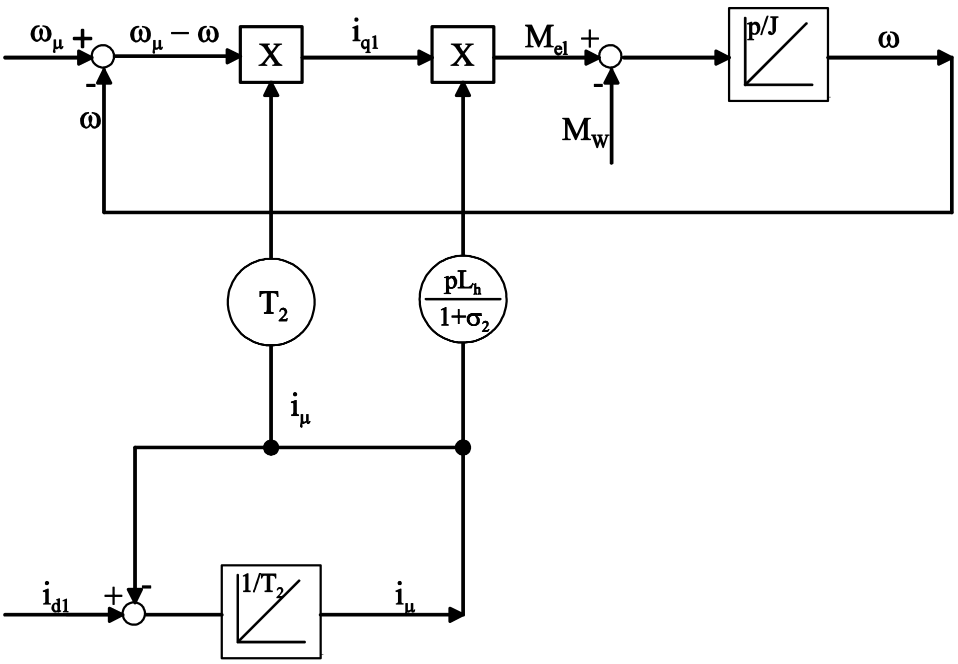


Fig. 40: induction machine, control strategy diagram

Same control strategy as used in DC machine applications:

Speed setpoint input with w_m , flux input with i_{d1}, instantaneous torque generation with i_{q1}:
$\rightarrow$ high-dynamic drive

3.5 Field-oriented control of induction machines with injected currents

As a matter of principle the intention is achieved. If the induction machine is controlled in field-oriented coordinates, it is required to know the angular position of the rotor flux.

It is not possible to measure any of the electric values in the rotor of squirrel cage machines - neither rotor currents nor rotor voltages. The measurement of the air-gap flux describes an approximation, which furthermore is expensive and susceptible to faults.
However magnitude and angular position of the rotor flux can be calculated from measured values of the stator currents and the speed, using the rotor equations of induction machines. This is called flux model, which is implemented on a microcontroller running in online operation.

$$T_2 \cdot \frac{di_m}{dt} + i_m = i_{d1} \tag{3.64}$$

$$\underbrace{\frac{i_{q1}}{T_2 \cdot i_m}}_{w_R} + w = w_m = \frac{da}{dt} \tag{3.65}$$

$$M_{el} = \frac{p \cdot L_h}{1 + s_2} \cdot i_m \cdot i_{q1} \tag{3.66}$$

$$\begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} = [T_a] \cdot \begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} \tag{3.67}$$

$$\begin{bmatrix} i_{A1} \\ i_{B1} \end{bmatrix} = [T_{32}] \cdot \begin{bmatrix} i_{u1} \\ i_{v1} \end{bmatrix} \tag{3.68}$$

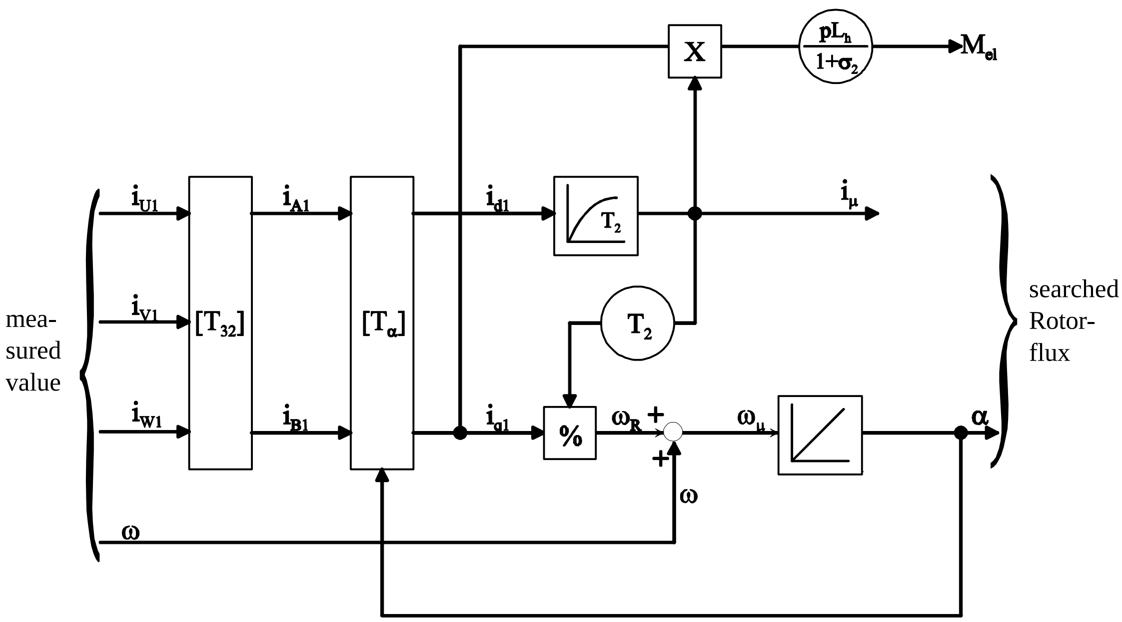


Fig. 41: induction machine, flux model in field-oriented coordinates

For this reason induction machines can be controlled in field-oriented coordinates with injected currents. The following picture shows the structure diagram of a complete drive system with speed-, torque- and flux controller.

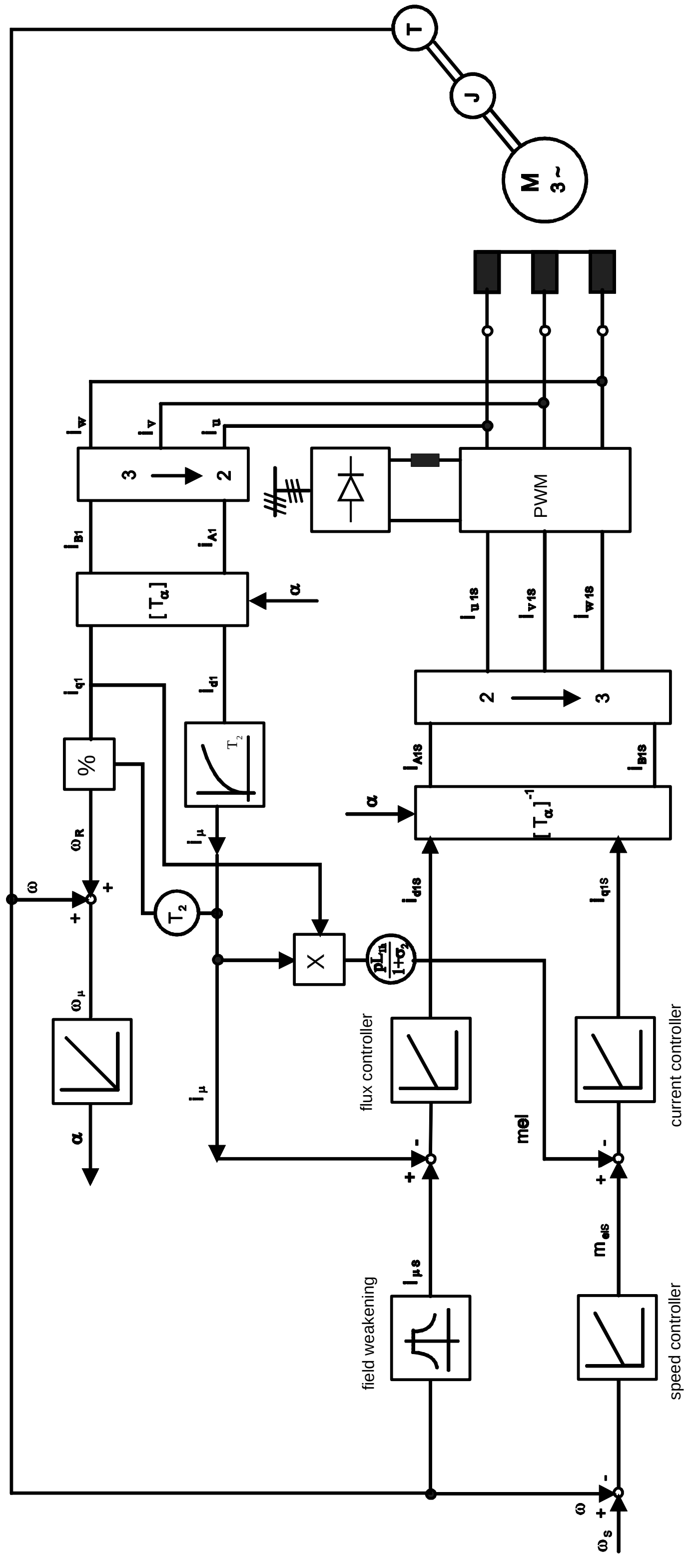


Fig. 42: entire drive system, structural diagram

The correct determination of i_m and a requires the knowledge of $T_2 = \dfrac{L_2}{R_2}$.

<u>Problem:</u> R_2 depends on temperature and L_2 depends on saturation effects.

The setpoint values for the flux and the torque are calculated from the speed setpoint input. In base speed operation the machine is driven with full flux and above the synchronous speed with field weakening (field control). Using a flux model with appropriate controller, the setpoint values for the transformed stator direct and quadrature currents are calculated. The three-phase current setpoint values for the pulse-controlled inverter result from an inverse transformation. Current i_m and position a are calculated online with the flux model, with knowledge of the measured stator current and speed values.

If the machine is in standstill at the instant $t = 0$ and a setpoint step-change $\frac{w}{w_1} = 1$ is enforced, the control unit injects the currents i_{u1}, i_{v1} and i_{w1} (i_{d1} and i_{q1} in the flux model after the transformation) with the pulse-controlled inverter in such a manner, that the magnetizing current i_m is built-up on its rated value with the rotor time constant $T_2 = \frac{L_2}{R_2}$. The machine accelerates nearly linear according to the adjusted quadrature current $i_{q1\max}$ during the acceleration time $T_J = \frac{J}{p} \cdot \frac{w_1}{M_{\max}}$.

There is no overshoot and there are no oscillations any more. → high dynamic drive

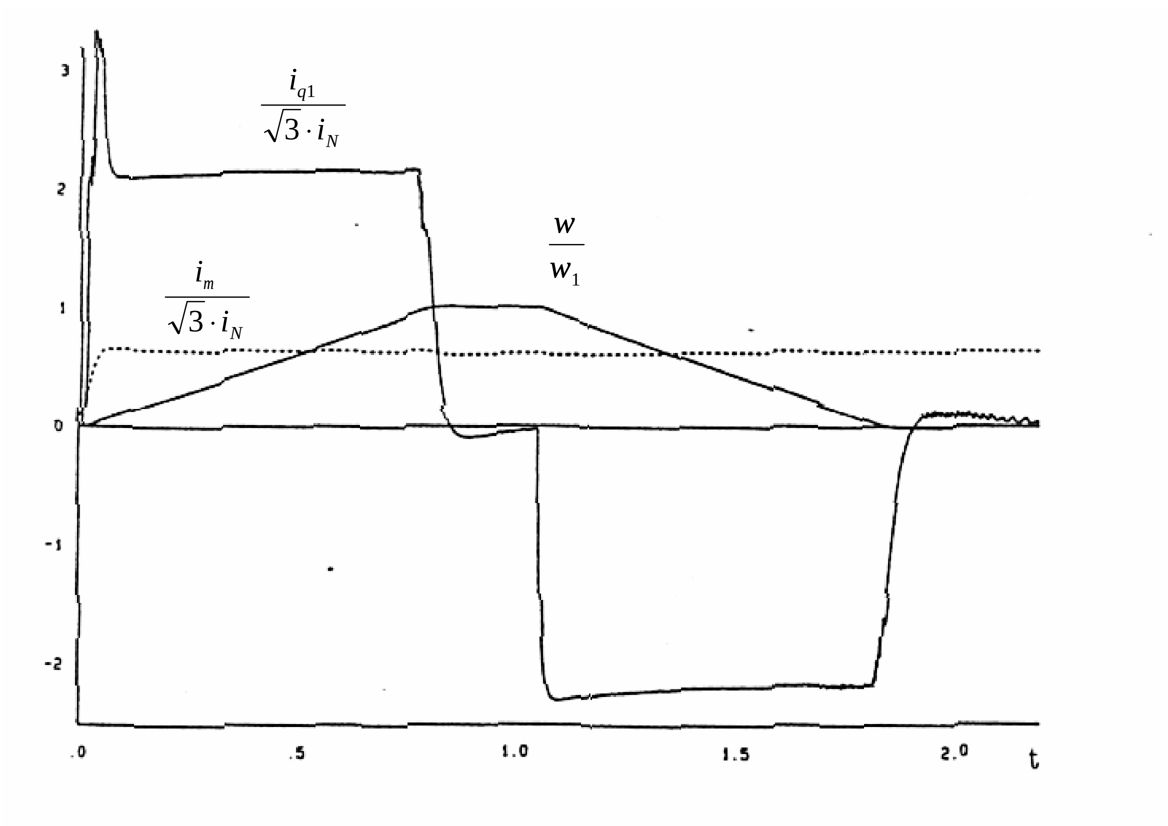


Fig. 43: simulation of an acceleration process and an afterwards braking

3.6 Steady-state operation using variable frequency and voltage converter

There are two possible fundamental operational performances:

<u>1.) Operation with constant stator flux-linkage</u>

The stator flux-linkages as well as the currents and voltages in steady-state operation are obtained by inverse transformation from rotating systems with $a(t) = w_1 \cdot t - \frac{p}{2}$:

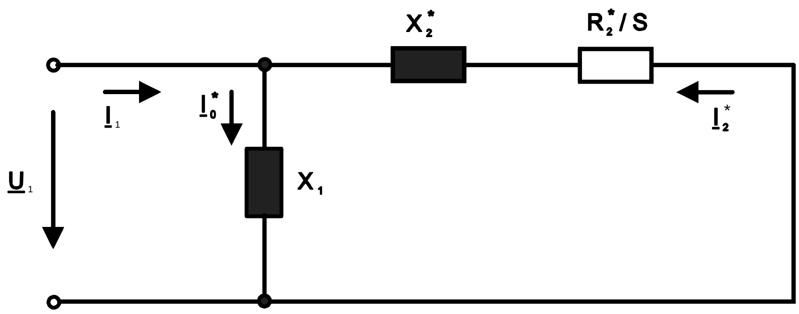
$$\underline{\Psi}_1 = \frac{\Psi_{q1}}{\sqrt{3}} - j \cdot \frac{\Psi_{d1}}{\sqrt{3}} = \frac{L_1 \cdot i_{q1} + L_h \cdot i'_{q2}}{\sqrt{3}} - j \frac{L_1 \cdot i_{d1} + L_h \cdot i'_{d2}}{\sqrt{3}} \tag{3.69}$$

$$= L_1 \cdot \underline{I}_{q1} + L_h \cdot \underline{I}'_{q2} + L_1 \cdot \underline{I}_{d1} + L_h \cdot \underline{I}'_{d2} \tag{3.70}$$

$$= L_1 \cdot \underline{I}_1 + L_h \cdot \underline{I}'_2 = L_1 \cdot \left(\underline{I}_1 + \frac{\underline{I}'_2}{1 + s_1} \right) \tag{3.71}$$

$$= L_1 \cdot \left(\underline{I}_1 + \underline{I}^*_2 \right) = L_1 \cdot \underline{I}^*_0 \tag{3.72}$$

For this purpose the equivalent circuit diagram (see script EM I, chapter 7.2) with $ü = \frac{w_1 \cdot x_1}{w_2 \cdot x_2} \cdot (1 + s_1)$ is suitable in particular.



$$\underline{I}^*_2 = \frac{\underline{I}'_2}{1 + s_1} \tag{3.73}$$

$$R^*_2 = R'_2 \cdot (1 + s_1)^2 \tag{3.74}$$

$$X^*_2 = \frac{s}{1 - s} \cdot X_1 \tag{3.73}$$

Fig. 44: induction machine, short-circuit ecd

In the equivalent circuit diagram (ecd) as shown in Fig. 44, with $R_1 = 0$ the current I^*_0 in the shunt arm has to be kept constant to achieve $\Psi_1 = const$.

$$\underline{\Psi}_1 = L_1 \cdot \underline{I}^*_0 = L_1 \cdot \frac{\underline{U}_1}{j \cdot X_1} = \frac{\underline{U}_1}{j \cdot w_1} = const \tag{3.76}$$

This means operation on a supply with $\frac{U_1}{f_1} = const$. The neglect of R_1 is only valid for high frequencies. For lower frequencies the supply voltage must be increased to compensate the resistance voltage drop at the stator resistance.

The locus diagram of the stator current is the well-known Heyland diagram of induction machines. The circle is practically parameterized with the rotor frequency.

$$\tan\left(j_{2}^{*}\right)=\frac{X_{2}^{*}}{R_{2}^{*}}\cdot s \sim s\cdot w_{1}=w_{2} \tag{3.77}$$

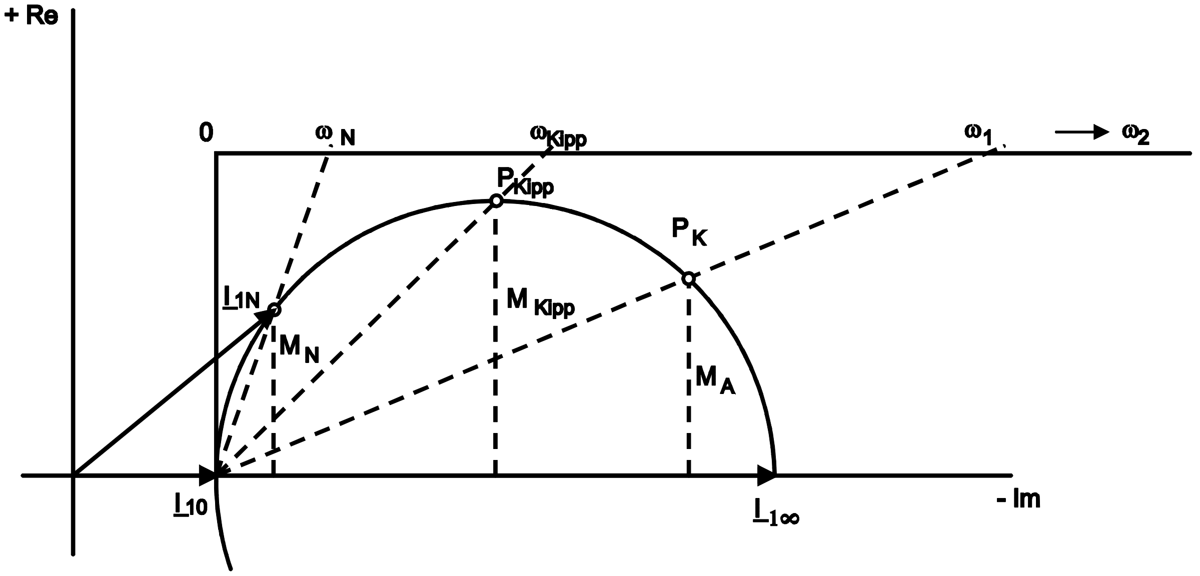


Fig. 45: induction machine, locus diagram (Heyland circuit)

The speed is adjusted with supply frequency and supply voltage $\frac{U_1}{f_1}=const$.

The peak value of the current is $I_{1\infty}$. The maximum torque is the breakdown torque M_{kipp} .

The relation between the breakdown slip and the rotor time constant is described by:

$$w_{kipp}=s_{kipp}\cdot w_{1}=\frac{R_{2}^{*}}{X_{2}^{*}}\cdot w_{1}=\frac{R_{2}^{'}\cdot\left(1+s_{1}\right)^{2}\cdot w_{1}}{w_{1}\cdot L_{1}\cdot\frac{s}{1-s}} \tag{3.78}$$

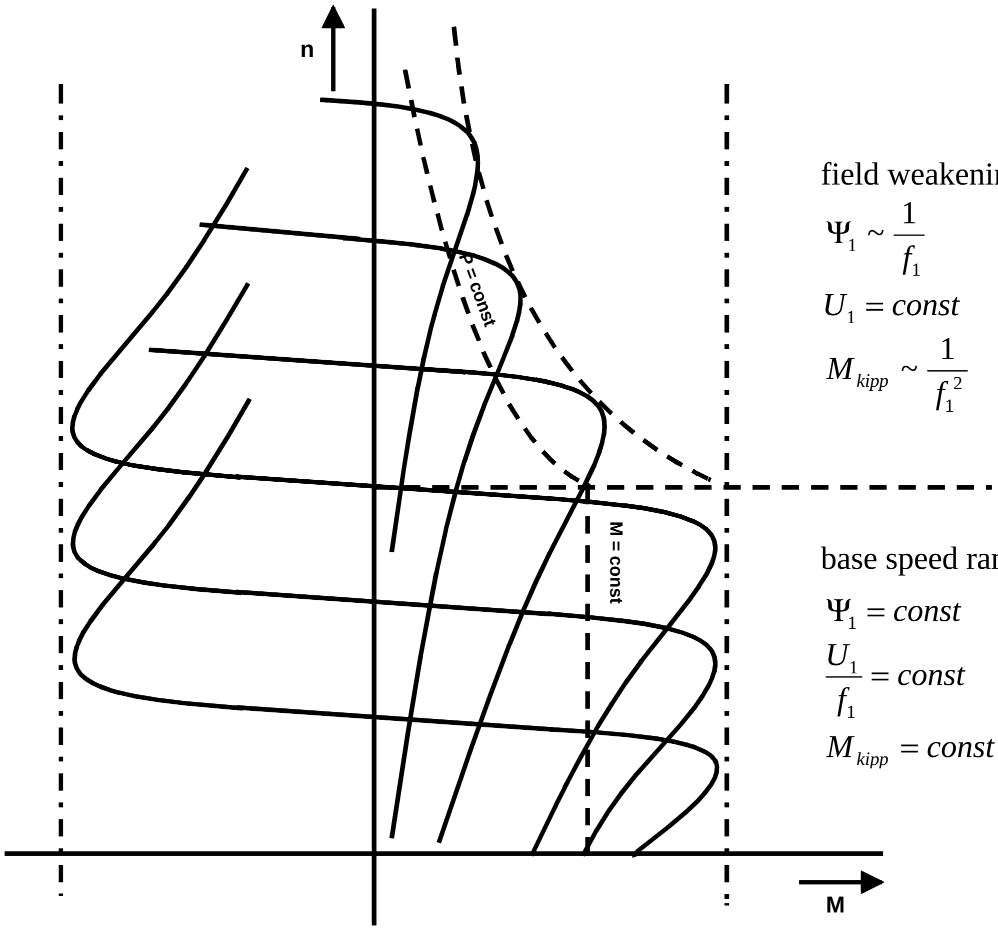
$$=\frac{R_{2}^{'}\cdot\left(1+s_{1}\right)^{2}}{\left(1+s_{1}\right)\cdot L_{h}\cdot s\cdot\left(1+s_{1}\right)\cdot\left(1+s_{2}\right)}=\frac{R_{2}^{'}}{s\cdot L_{2}^{'}}=\frac{1}{s\cdot T_{2}} \tag{3.79}$$

The *Kloss* equation describes the torque as a function of rotor frequency.

$$\frac{M}{M_{kipp}}=\frac{2}{\frac{s}{s_{kipp}}+\frac{s_{kipp}}{s}}=\frac{2}{\frac{w_{2}}{w_{kipp}}+\frac{w_{kipp}}{w_{2}}} \tag{3.80}$$

For different values of the supply frequency a family of characteristics is obtained, which appears quite similar to the characteristic of DC machines for $w_2 < w_{kipp}$.

For $\Psi_1 = \Psi_{10}$ (no-load flux-linkage) is $I_1 = I_{0N} = \dfrac{U_{1N,Str}}{X_1}$ (no-load current).



field weakening range

$\Psi_1 \sim \dfrac{1}{f_1}$

$U_1 = const$

$M_{kipp} \sim \dfrac{1}{f_1^2}$

base speed range

$\Psi_1 = const$

$\dfrac{U_1}{f_1} = const$

$M_{kipp} = const$

Fig. 46: induction machine, torque-speed diagram

<u>2.) Operation with constant rotor flux-linkage</u>

Again the rotor flux-linkage as well as the stator flux-linkage are obtained by inverse transformation from the rotating system with $a = w_1 \cdot t - \dfrac{p}{2}$.

Practically the rotor flux-linkage for field-oriented operation is to be chosen. With that the induction machine is expected to behave like a separately excited DC machine.

$$\underline{\Psi}_2' = \frac{\Psi_{q2}'}{\sqrt{3}} - j \cdot \frac{\Psi_{d2}'}{\sqrt{3}} = \frac{0}{\sqrt{3}} - j \cdot \frac{L_h \cdot i_m}{\sqrt{3}} \tag{3.81}$$

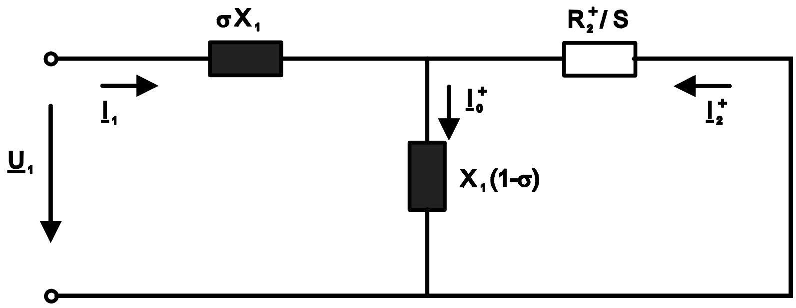
We define:

$$\underline{I}_0^+ = -j \cdot \frac{i_m}{\sqrt{3}} \tag{3.82}$$

and divide Ψ_2' by $(1+s_2)$

$$\frac{\underline{\Psi}_2'}{(1+s_2)} = \frac{L_h \cdot \underline{I}_0^+}{(1+s_2)} = \frac{(1+s_1)\cdot L_h \cdot \underline{I}_0^+}{(1+s_1)\cdot(1+s_2)} = (1-s)\cdot L_1 \cdot \underline{I}_0^+ \tag{3.83}$$

For this purpose the equivalent circuit diagram (ecd) with $\ddot{u} = \frac{w_1 \cdot x_1}{w_2 \cdot x_2} \cdot \frac{1}{(1+s_2)}$ is reasonable.



$$\underline{I}_2^+ = \underline{I}_2^{'} \cdot (1+s_2) \quad (3.84)$$

$$R_2^+ = \frac{R_2^{'}}{(1+s_2)^2} \quad (3.85)$$

$$\underline{I}_0^+ = \underline{I}_1 + \underline{I}_2^{'} \quad (3.84)$$

Fig. 47: induction machine, ecd for constant rotor flux linkage

In the equivalent network with $R_1 = 0$ the current I_0^+ in the shunt arm has to be kept constantly to achieve $\Psi_2^{'} = const$. This is <u>*field-oriented operation*</u>. The voltage drop at $s \cdot L_1$ has to be compensated load-dependent:

$$\frac{\underline{U}_1}{w_1} = j \cdot s \cdot L_1 \cdot \underline{I}_1 + j \cdot (1-s) \cdot L_1 \cdot \underline{I}_0^+ \quad (3.87)$$

For this purpose the stator has to be dimensioned adequate, to avoid saturation phenomena.

Again the neglect of R_1 is only valid for high frequencies. In the case of low frequencies the supply voltage must be increased to compensate the resistance voltage drop at the stator resistance R_1.

The locus diagram of the stator current I_1 dependent on the rotor frequency w_2 is now a line for $\Psi_2^{'} = const$ and the stator voltage U_1 is load-dependent.

$$\underline{U}_1 = j \cdot s \cdot X_1 \cdot \underline{I}_1 + j \cdot (1-s) \cdot X_1 \cdot \underline{I}_0^+ \quad (3.88)$$

$$\underline{I}_0^+ = \underline{I}_1 + \underline{I}_2^+ \quad (3.89)$$

$$\underline{I}_2^+ \cdot \frac{R_2^+}{s} + \underline{I}_0^+ \cdot j \cdot (1-s) \cdot X_1 = 0 \quad (3.90)$$

For I_1 follows:

$$\underline{I}_1 = \underline{I}_0^+ - \underline{I}_2^+ = \underline{I}_0^+ + j \cdot \frac{\underline{I}_0^+ \cdot (1-s) \cdot X_1}{R_2^+ / s} = \underline{I}_0^+ \cdot \left(1 + j \cdot \frac{(1-s) \cdot w_1 \cdot L_1}{\frac{R_2^+}{w_2 / w_1}} \right) \quad (3.91)$$

$$= \underline{I}_0^+ \cdot \left(1 + j \cdot \frac{(1+s_1) \cdot L_h \cdot w_2}{(1+s_1) \cdot (1+s_2) \cdot \frac{R_2^{'}}{(1+s_2)^2}} \right) = \underline{I}_0^+ \cdot (1 + j \cdot w_2 \cdot T_2) = \underline{I}_0^+ \cdot \left(1 + j \cdot \frac{w_2}{s \cdot w_{kipp}} \right) \quad (3.92)$$

For $\Psi_2^{'} = \Psi_{20}^{'}$ (no-load flux linkage) is (3.93)

$$I_0^+ = I_{0N} = \frac{U_{1N,Str}}{X_1} \quad \text{(no-load current).} \qquad (3.94)$$

and the stator current ensues to:

$$I_1 = \sqrt{I_m^2 + I_{Wirk}^2}\ . \qquad (3.93)$$

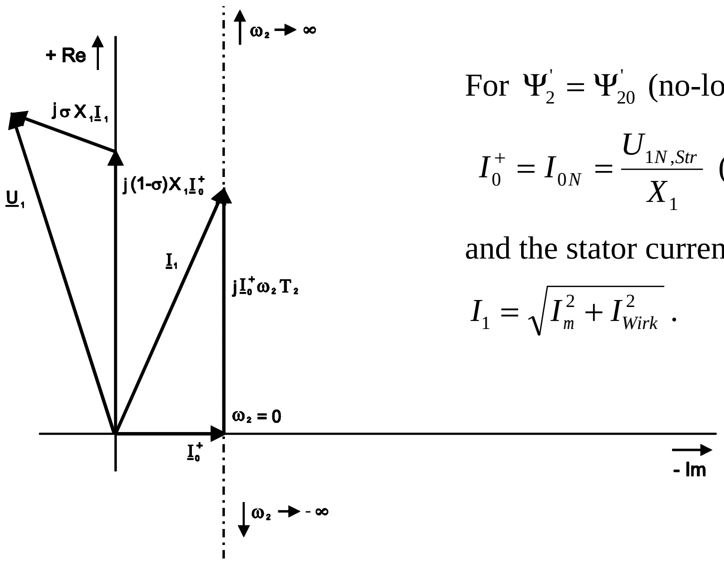


Fig. 48: induction machine, phasor diagram

Specific points:

- $w_2 = 0: \quad \underline{I}_1 = \underline{I}_{0N}^+ = \frac{\underline{U}_{1N,Str}}{j \cdot X_1}$ no-load flux linkage (3.96)
- $w_2 \to \infty: \ \underline{I}_1 \to \infty$ stator current (unlimited) (3.97)

i) The active current is:

$$I_{1W} = \text{Re}\{\underline{I}_1\} = \frac{I_{0N}}{s} \cdot \frac{w_2}{w_{kipp}} = I_\infty \cdot \frac{w_2}{w_{kipp}} \qquad (3.98)$$

which means that the rotor frequency adjusts itself load-dependent.

ii) The reactive current is constant and equal to the no-load current, the power factor $\cos j$ is is improved (=increased) compared to $\Psi_1 = const$.

$$I_{1B} = \text{Im}\{\underline{I}_1\} = I_{0N} \qquad (3.99)$$

For $w_2 = w_{kipp}$ is $I_{1W} = I_\infty$.

Breakdown slip and current limitation do not apply anymore.

The torque is calculated with the induced voltage and the active current ($\Rightarrow$ compare to DC machines).

$$M = \frac{3 \cdot p}{w_1} \cdot U_i \cdot I_{1W} = \frac{3 \cdot p}{w_1} \cdot (1 - s) \cdot X_1 \cdot I_{0N} \cdot \frac{I_{0N}}{s} \cdot \frac{w_2}{w_{kipp}} \qquad (3.100)$$

$$= \frac{3 \cdot p}{w_1} \cdot \frac{U_{1N}^2}{X_1 \cdot \frac{s}{1-s}} \cdot \frac{w_2}{w_{kipp}} = 2 \cdot M_{kipp} \cdot \frac{w_2}{w_{kipp}} \qquad (3.101)$$

For $w_2 = w_{kipp}$ is $M = 2 \cdot M_{kipp}$

The speed-torque characteristic is now a (declining) straight line and looks like the characteristic of separately excited DC machines.

$$w_2 = \frac{M \cdot w_{kipp}}{2 \cdot M_{kipp}} \tag{3.102}$$

$$n = n_1 - n_2 = \frac{f_1}{p} - \frac{w_2}{2 \cdot p \cdot p} = \frac{f_1}{p} - \frac{f_{kipp}}{p} \cdot \frac{M}{2 \cdot M_{kipp}} \tag{3.103}$$

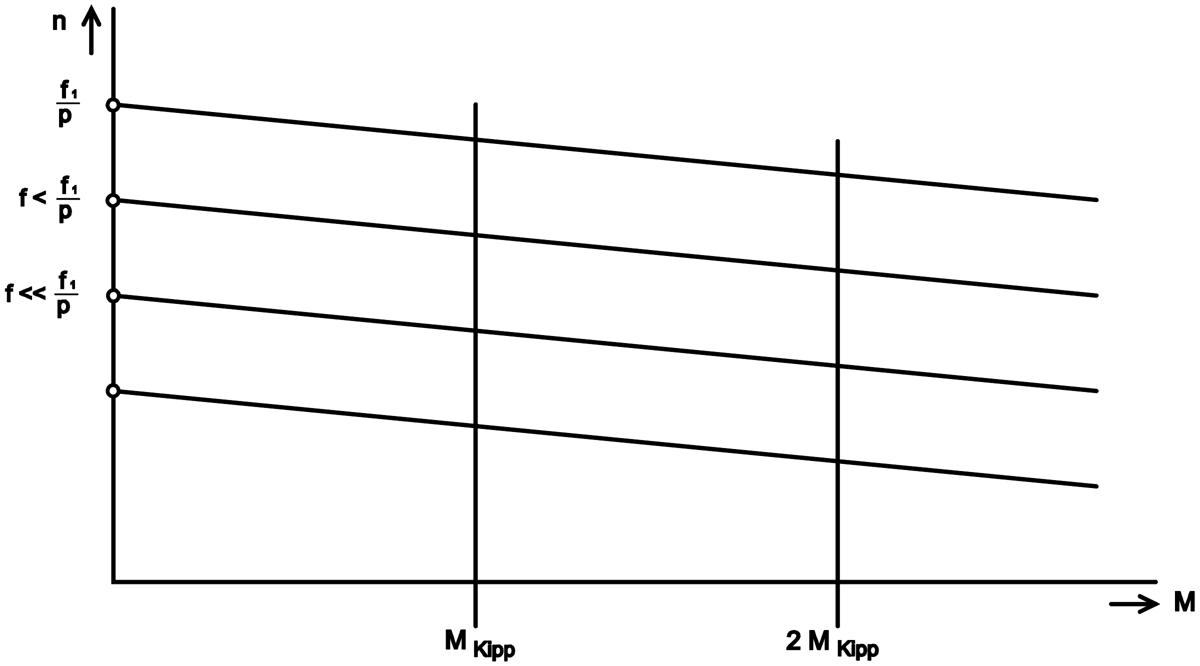


Fig. 49: induction machine, torque-speed characteristics

The speed drop in the load case is halved for $\Psi_2' = const$ compared to the case of $\Psi_1 = const$.

A summarizing comparison and conclusion of both described methods

1.) constant *stator* flux-linkage

2.) constant *rotor* flux-linkage

is shown in the table on the next page.

constant *stator* flux-linkage	constant *rotor* flux-linkage
$\Psi_1 = const = \Psi_{10}$	$\dfrac{\Psi_2'}{1+s_2} = const = \dfrac{\Psi_{20}'}{1+s_2}$
here: $I_{0N} = \dfrac{U_N / \sqrt{3}}{w_{1N} \cdot L_1}$	
ecd with $ü = \dfrac{w_1 \cdot x_1}{w_2 \cdot x_2} \cdot (1+s_1)$	**ecd with** $ü = \dfrac{w_1 \cdot x_1}{w_2 \cdot x_2} \cdot \dfrac{1}{(1+s_2)}$
$L_1 \cdot I_{0N} = const$	$(1-s) \cdot L_1 \cdot I_{0N} = const$
(Does not have to be no-load flux-linkage. Rated flux-linkage is also possible.)	
$\dfrac{\underline{U}_1}{w_1} - \dfrac{R_1}{w_1} \cdot \underline{I}_1 = const$	$\dfrac{\underline{U}_1}{w_1} - \dfrac{R_1}{w_1} \cdot \underline{I}_1 - j \cdot s \cdot L_1 \cdot \underline{I}_1 = const$
for $R_1 = 0$**:** $\dfrac{U_1}{f_1} = const$	**for** $R_1 = 0$**:** $\dfrac{U_1}{f_1} - 2 \cdot p \cdot j \cdot s \cdot L_1 \cdot \underline{I}_1 = const$
locus diagram: circle	**locus diagram: line**
$\underline{I}_1 = \underline{I}_{0N} \cdot \left(1 + \dfrac{j \cdot \dfrac{1-s}{s}}{\dfrac{w_{kipp}}{w_2} + j} \right)$	$\underline{I}_1 = \underline{I}_{0N} \cdot (1 + j \cdot w_2 \cdot T_2)$
speed control with w_1 w_2 **adjusts itself due to load**	
$M = M_{kipp} \cdot \dfrac{2}{\dfrac{w_{kipp}}{w_2} + \dfrac{w_2}{w_{kipp}}}$	$M = 2 \cdot M_{kipp} \cdot \dfrac{w_2}{w_{kipp}}$
if $w_2 = w_{kipp}$ **follows**	
$M = M_{kipp} \quad (\hat{=} M_{\max})$	$M = 2 \cdot M_{kipp} \quad (M_{\max} \to \infty)$
$I_{1W} = \dfrac{I_ø}{2}$	$I_{1W} = I_\infty \quad (\hat{=} i_{B1} / \sqrt{3})$
$I_{1B} = \dfrac{I_ø}{2} + I_0$	$I_{1B} = I_0 \quad (\hat{=} i_{A1} / \sqrt{3})$
may become unstable, **operation at variable voltage and frequency supply**	**may not become unstable,** **better power factor** $\cos j$ **,** **DC machine behavior,** **field-oriented control**

3.7 Field-oriented control of induction machines with applied voltages

Up to now the field-oriented control of induction machines was deduced using the simplifying assumption, that a current injecting power converter with high switching frequency, with fast control unit and sufficient voltage reserve is available. This is the case for servo drives with transistorized frequency converters and switching frequencies up to 20 kHz in the kW power range. For larger drives GTO-pulse-controlled inverters with variable-voltage link and switching frequencies lower than 1 kHz are used. In the discussed case the assumption is no longer fulfilled, so the stator voltage equations of the induction machine have to be taken into consideration.

After pasting the rotor currents from chapter 3.4, the stator flux-linkages are:

$$\Psi_{d1} = (1+s_1)\cdot L_{1h}\cdot i_{d1} + L_{1h}\cdot i'_{d2} = (1+s_1)\cdot L_{1h}\cdot i_{d1} + \frac{L_{1h}}{1+s_2}\cdot (i_m - i_{d1}) \tag{3.104}$$

$$= (1+s_1)\cdot L_{1h}\cdot i_{d1}\cdot\left(1-\frac{1}{(1+s_1)(1+s_2)}\right) + \frac{L_{1h}}{(1+s_2)}\cdot i_m = \frac{L_{1h}}{1+s_2}\cdot i_m + s\cdot L_1\cdot i_{d1} \tag{3.105}$$

$$\Psi_{q1} = (1+s_1)\cdot L_{1h}\cdot i_{q1} + L_{1h}\cdot i'_{q2} = (1+s_1)\cdot L_{1h}\cdot i_{q1} - \frac{L_{1h}}{1+s_2}\cdot i_{q1} \tag{3.106}$$

$$= s\cdot L_1\cdot i_{q1} \tag{3.107}$$

Now stator voltage equations can be converted:

$$u_{d1} = R_1\cdot i_{d1} + \frac{d\Psi_{d1}}{dt} - w_m\cdot\Psi_{q1} = R_1\cdot i_{d1} + \frac{L_{1h}}{1+s_2}\cdot\frac{di_m}{dt} + s\cdot L_1\cdot\frac{di_{d1}}{dt} - w_m\cdot s\cdot L_1\cdot i_{q1} \tag{3.108}$$

$$u_{q1} = R_1\cdot i_{q1} + \frac{d\Psi_{q1}}{dt} + w_m\cdot\Psi_{d1} = R_1\cdot i_{q1} + s\cdot L_1\cdot\frac{di_{q1}}{dt} + w_m\cdot\frac{L_{1h}\cdot i_m}{1+s_2} + w_m\cdot s\cdot L_1\cdot i_{d1} \tag{3.109}$$

With the definition of the stator time constant T_1

$$T_1 = \frac{(1+s_1)\cdot L_{1h}}{R_1} \tag{3.110}$$

the stator voltage equations in field-oriented coordinates are obtained:

$$s\cdot T_1\cdot\frac{di_{d1}}{dt} + i_{d1} = \frac{1}{R_1}\cdot\left(u_{d1} + w_m\cdot s\cdot L_1\cdot i_{q1} - (1-s)\cdot L_1\cdot\frac{di_m}{dt}\right) \tag{3.111}$$

$$s\cdot T_1\cdot\frac{di_{q1}}{dt} + i_{q1} = \frac{1}{R_1}\cdot\left(u_{q1} - w_m\cdot s\cdot L_1\cdot i_{d1} - (1-s)\cdot L_1\cdot w_m\cdot i_m\right) \tag{3.112}$$

The stator voltage equations complete the machine model with the co-action of the stator voltages and the stator currents.

Concerning the components of the stator currents the induction machine behaves like a first-order time-delay element with the time constant $\sigma \cdot T_1$ and the gain $\frac{1}{R_1}$. The components of the stator currents are coupled by the right side of the equations. $w_m \cdot \sigma \cdot L_1 \cdot i_{d1}$ and $w_m \cdot \sigma \cdot L_1 \cdot i_{q1}$ are rotary induced voltages, caused by the current in the particular quadrature axis. $(1-\sigma) \cdot L_1 \cdot \frac{di_m}{dt}$ is a transformer voltage, which is caused by the change of the magnetizing current. $(1-\sigma) \cdot L_1 \cdot w_m \cdot i_m$ is the rotary induced voltage of the magnetizing field.

Both control systems are coupled by the stator currents and are not independent from each other. However a decoupling is desired in such a manner, that the current controllers can be adjusted independently. This is realized by adding compensating voltages with negative sign to the controller output voltages u_{Rd} and u_{Rq}, so that the coupling voltages disappear. So the controllers see decoupled controlled systems. Assumption for the compensation: the rotor flux linkage needs to be constant, i.e. $\frac{di_m}{dt} = 0$.

$$\sigma \cdot T_1 \cdot \frac{di_{d1}}{dt} + i_{d1} = \frac{u_{Rd}}{R_1} \tag{3.113}$$

$$\sigma \cdot T_1 \cdot \frac{di_{q1}}{dt} + i_{q1} = \frac{u_{Rq}}{R_1} \tag{3.114}$$

$$u_{Rd} = u_{d1} - w_m \cdot \sigma \cdot L_1 \cdot i_{q1} \tag{3.115}$$

$$u_{Rq} = u_{q1} + w_m \cdot \sigma \cdot L_1 \cdot i_{d1} + (1-\sigma) \cdot L_1 \cdot w_m \cdot i_m \tag{3.116}$$

The compensating voltages are generated in a decoupling network (Fig. 50).

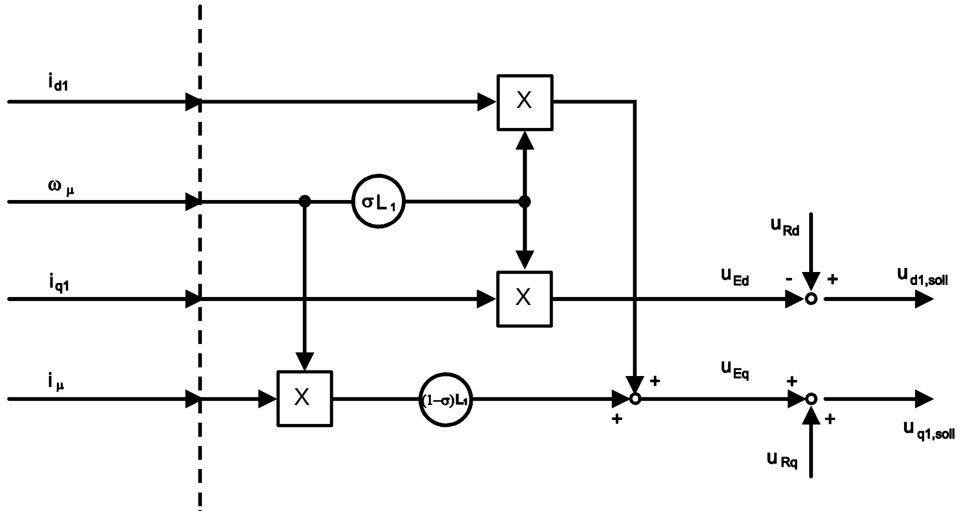


Fig. 50: induction machine, control strategies, decoupling network

Fig. 51 shows the complete circuit of a field-oriented induction machine with voltage injecting actuator with pulse-width modulation. A cascade control is used for the direct- and the quadrature-current, which can be adjusted independently with the aid of a decoupling network. The actual values of magnitude and position of the rotor flux, which are required for the control, are determined with a flux model (see next page for figure).

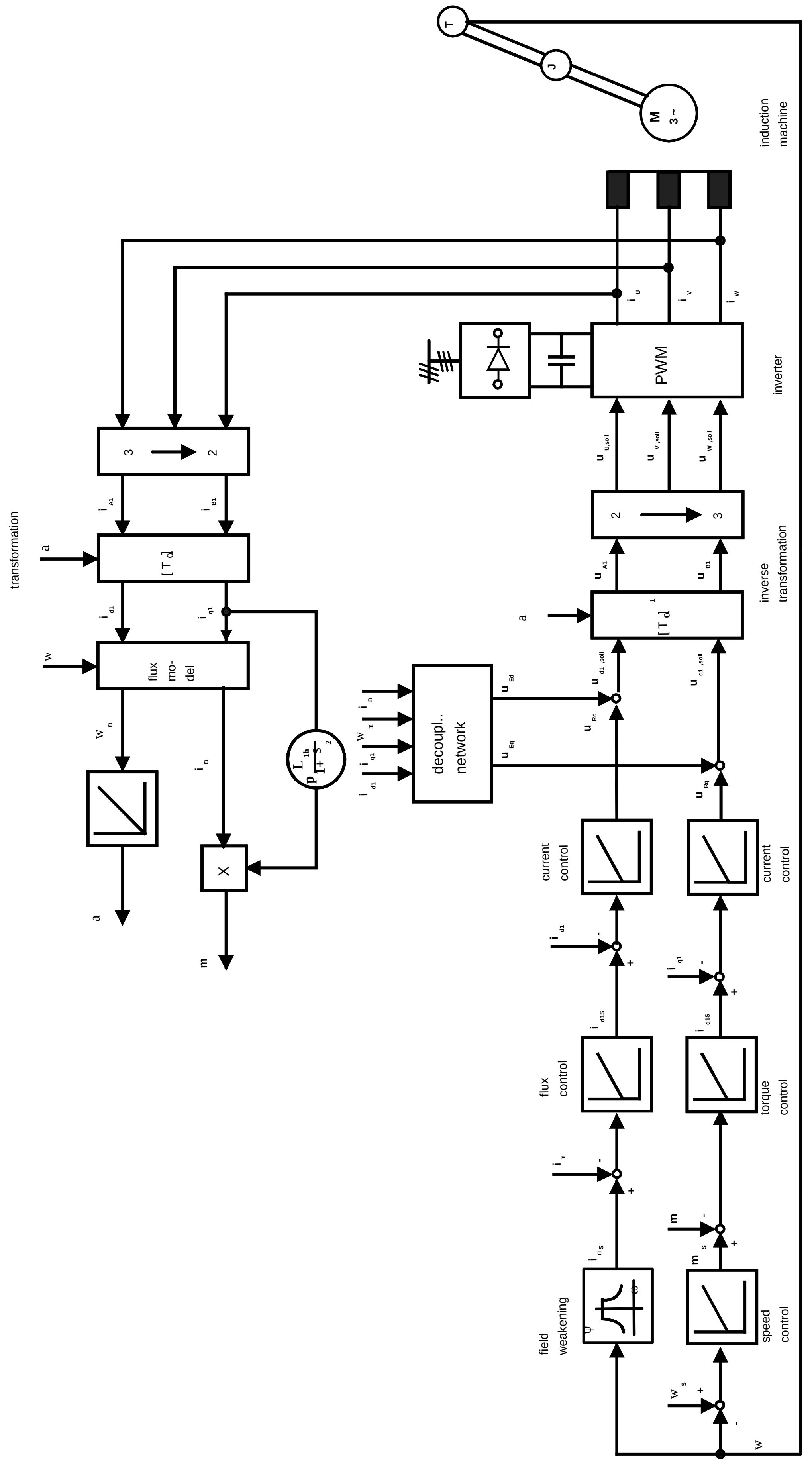


Fig. 51: field-oriented induction machine, voltage injecting actuator, pulse-width modulation

4 Synchronous machine

4.1 Dynamic system of equations

In this section focus is put on salient-pole synchronous machines with distinctive magnetic biaxiality and definitive damper winding. The following features have to be taken into consideration:

- permeances of direct- and quadrature-axis differ, $X_d \neq X_q$
- transformation has to be carried out on the asymmetric part of the system, which means the rotor. $\frac{da}{dt} = \frac{dg}{dt}$
- damper winding takes effect as well in the direct-axis as in the quadrature-axis. Therefore a short-circuited direct-axis damper-winding and a short-circuited quadrature-axis damper-winding has to be taken into account additionally to the excitation winding.
- the generator reference-arrow system (EZS) is usually chosen for generator operation. The reference angle of the rotating coordinate system equals $a_0 = -\frac{p}{2} + J$

Differing from the previous section we now omit the indices 1 and 2 for the stator and the rotor. According to the literature lower case letters are used for the stator and capital letters for the rotor. The conversion of the rotor on the number of the stator windings is retained and indicated with a dash. For this reason the following denominations are obtained for the real machine and for the transformed machine in the biaxial system:

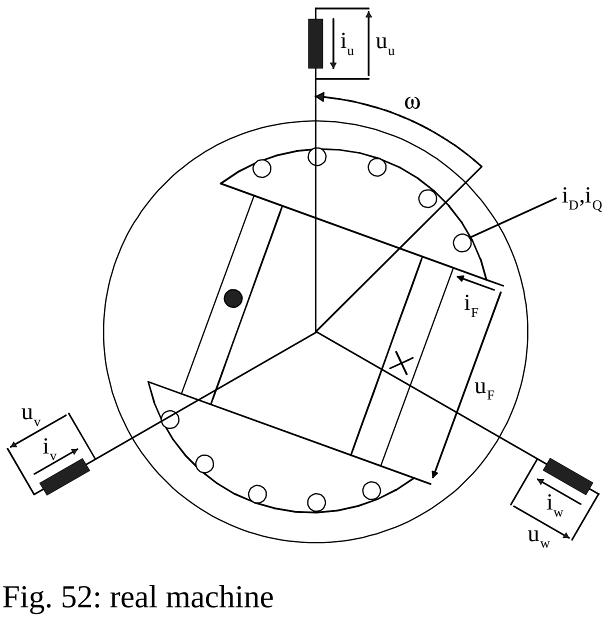


Fig. 52: real machine

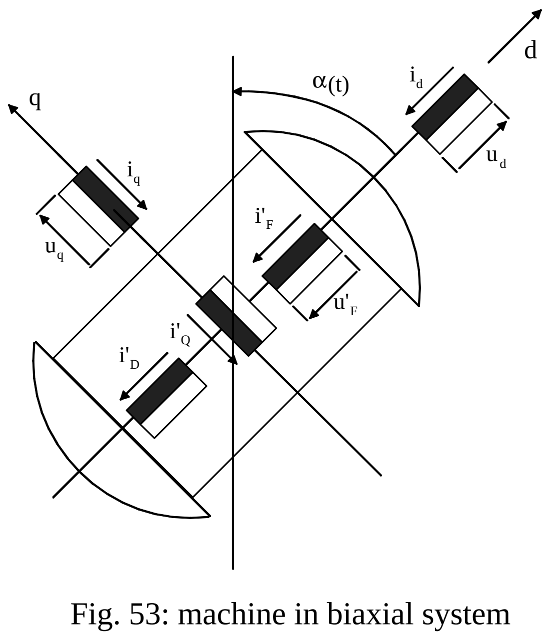


Fig. 53: machine in biaxial system

note: stator in EZS
rotor in VZS

d: stator direct axis, q: stator quadrature axis
D', Q' according for damper windings
F' index for rotor direct axis

In the equivalent winding system all the windings are separated in magnetizing in direct- and quadrature axis. Five voltage equations and one torque equation apply:

$$-u_d = R_1 \cdot i_d + \frac{d\Psi_d}{dt} - w \cdot \Psi_q \tag{4.1}$$

$$-u_q = R_1 \cdot i_q + \frac{d\Psi_q}{dt} + w \cdot \Psi_d \tag{4.2}$$

$$u_F^{'} = R_F^{'} \cdot i_F^{'} + \frac{d\Psi_F^{'}}{dt} \tag{4.3}$$

$$0 = R_D^{'} \cdot i_D^{'} + \frac{d\Psi_D^{'}}{dt} \tag{4.4}$$

$$0 = R_Q^{'} \cdot i_Q^{'} + \frac{d\Psi_Q^{'}}{dt} \tag{4.5}$$

$$M_{el} = p \cdot \left(\Psi_d \cdot i_q - \Psi_q \cdot i_d\right) = \frac{J}{p} \cdot \frac{dw}{dt} - M_A \tag{4.6}$$

In generator operation the torque on the shaft is not driving but braking:

$$M_A = -M_W \tag{4.7}$$

The following relations result for 5 accordant flux linkages (summarized in a matrix):

$$\begin{bmatrix} \Psi_d \\ \Psi_F^{'} \\ \Psi_D^{'} \\ \Psi_q \\ \Psi_Q^{'} \end{bmatrix} = \begin{bmatrix} L_d & L_{hd} & L_{hd} & 0 & 0 \\ L_{hd} & L_F^{'} & L_{hd} & 0 & 0 \\ L_{hd} & L_{hd} & L_D^{'} & 0 & 0 \\ 0 & 0 & 0 & L_q & L_{hq} \\ 0 & 0 & 0 & L_{hq} & L_Q^{'} \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_F^{'} \\ i_D^{'} \\ i_q \\ i_Q^{'} \end{bmatrix} \tag{4.8 a-e}$$

Utilized resistances and inductances are denominated as follows:

- stator direct axis: R_1 $L_d = L_{1s} + L_{hd}$
- stator quadrature axis: R_1 $L_q = L_{1s} + L_{hq}$
- excitation winding: $R_F^{'}$ $L_F^{'} = L_{Fs}^{'} + L_{hd}$
- damper winding direct-axis: $R_D^{'}$ $L_D^{'} = L_{Ds}^{'} + L_{hd}$
- damper winding quadr. axis: $R_Q^{'}$ $L_Q^{'} = L_{Qs}^{'} + L_{hq}$

(the last three: converted to the number of stator windings)

The matrix of the inductances is symmetric because the transformation is power-invariant.

The transformation of the stator voltages and currents results in:

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = [T_a] \cdot [T_{32}] \cdot \begin{bmatrix} u_u \\ u_v \end{bmatrix} \tag{4.9}$$

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = [T_a] \cdot [T_{32}] \cdot \begin{bmatrix} i_u \\ i_v \end{bmatrix} \tag{4.10}$$

The rotor quantities do not need to be transformed, because they are already separated in 2 perpendicular axes rotating with $\frac{dg}{dt}$.

So that the salient-pole synchronous-machine is completely described with

- five voltage equations,
- one torque equation and
- five flux-linkage equations

in a two-axis system rotating with synchronous angular speed.

The non-salient-pole machine (*cylindrical-rotor machine*) can be understood as a special case featuring $L_d = L_q$.

If damper windings are omitted and permanent magnets are treated as an injected magnetic field, the permanent-field synchronous machine with rotor position encoder, the so called EC-motor, is also described.

The system of differential equations is nonlinear and therefore it can only be solved completely using numerical methods on a computer. Only if the speed is defined to be constant, an analytic solution can be found.

As it was formulated in the requirements, the zero phase-sequence system is neglected. But if the star point is connected and the load is not symmetrical, then there are a star point current and a star point voltage:

$$u_{St} = u_u + u_v + u_w = 3 \cdot u_0 \neq 0 \tag{4.11}$$

If the zero phase-sequence system is separated from the unsymmetrical system, a symmetrical system ensues, which can handled as before:

$$u_u^* = u_u - u_0, \qquad u_v^* = u_v - u_0, \qquad u_w^* = u_w - u_0 \tag{4.12 a-c}$$

For the zero phase-sequence system additional voltage- and flux-linkage equations are obtained, which can be solved separately:

$$-u_0 = R_0 \cdot i_0 + \frac{d\Psi_0}{dt} \qquad \text{(EZS)} \tag{4.13}$$

$$\Psi_0 = L_0 \cdot i_0 \tag{4.14}$$

L_0 and R_0 can be measured, if all the three winding phases are supplied in phase:

$$R_0 = 3 \cdot R_1, \qquad L_0 = 3 \cdot L_{1s} \tag{4.15 a,b}$$

After the inverse transformation of the solution of the symmetrical system, the solution is added to the zero phase-sequence system solution:

$$i_u = i_u^* + i_0, \qquad i_v = i_v^* + i_0, \qquad i_w = i_w^* + i_0 \tag{4.16}$$

4.2 Steady-state operation of salient-pole machines at mains power supply

In steady-state operation the flux-linkages in the rotating system are constant, i.e. $\frac{d\Psi}{dt} = 0$ and the rotational speed is constant, i.e. $\frac{dw}{dt} = 0$.

Therefore the voltage equations and the torque equation are decoupled. The stator resistance is neglected, i.e. $R_1 = 0$. For this reason the system of equations is simplified as follows:

$$u_d = w \cdot \Psi_q \tag{4.17}$$

$$u_q = -w \cdot \Psi_d \tag{4.18}$$

$$u_F^{'} = R_F^{'} \cdot i_F^{'} \tag{4.19}$$

$$i_D^{'} = 0 \tag{4.20}$$

$$i_Q^{'} = 0 \tag{4.21}$$

$$M_A = p \cdot \left(\Psi_q \cdot i_d - \Psi_d \cdot i_q\right) \tag{4.22}$$

$$\Psi_d = L_d \cdot i_d + L_{hd} \cdot i_F^{'} \tag{4.23}$$

$$\Psi_q = L_q \cdot i_q \tag{4.24}$$

The other flux-linkages are not taken care of, the voltage equation for the excitation winding is trivial and the damper currents in steady-state operation are equal zero. Pasting the flux-linkages results in:

$$u_d = w \cdot L_q \cdot i_q \tag{4.25}$$

$$u_q = -w \cdot L_d \cdot i_d - w \cdot L_{hd} \cdot i_F^{'} \tag{4.26}$$

$$M_A = \frac{p}{w} \cdot \left(u_d \cdot i_d + u_q \cdot i_q\right) \tag{4.27}$$

For the inverse transformation of the system rotating with $\frac{da}{dt} = \frac{dg}{dt}$ the choice of the integration constant a_0 is still free. Practically we choose for the synchronous machine:

$$a_0 = -\frac{p}{2} + J \qquad (4.28)$$

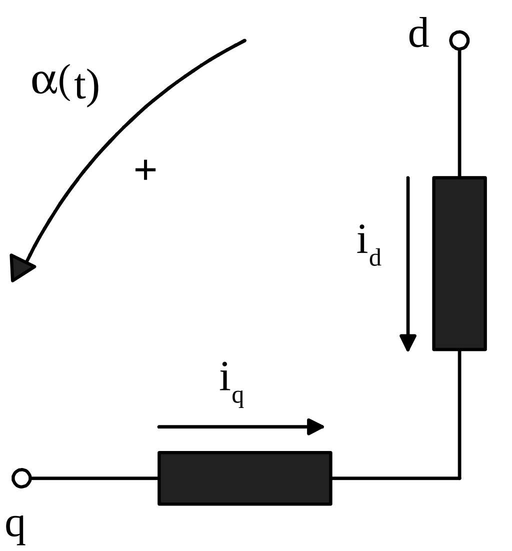


Fig. 54: coordinate system

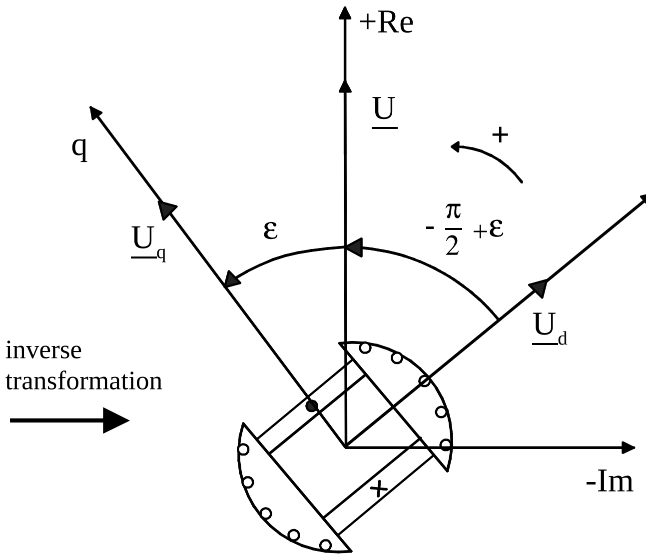


Fig. 55: complex plane

$$\underline{U} = \frac{u_d}{\sqrt{3}} \cdot e^{ja_0} + j\frac{u_q}{\sqrt{3}} \cdot e^{ja_0} = \underline{U}_d + \underline{U}_q \qquad (4.29)$$

Then the rotor axis, in which the machine is magnetized (direct axis) lags behind with an angle of $-\frac{p}{2} + J$ and the quadrature axis (the axis of the synchronous generated voltage) is leading with an angular displacement of J. As desired the terminal voltage coincides with the real axis.

Inverse transformation:

$$\underline{U}_d = \frac{w \cdot L_q \cdot i_q}{\sqrt{3}} \cdot e^{-j\frac{p}{2}} \cdot e^{jJ} = -jw \cdot L_q \cdot \frac{i_q}{\sqrt{3}} \cdot e^{jJ} \qquad (4.30)$$

$$\underline{U}_q = j\frac{-w \cdot L_d \cdot i_d - w \cdot L_{hd} \cdot i_F^{'}}{\sqrt{3}} \cdot e^{-j\frac{p}{2}} \cdot e^{jJ}$$

$$= -jw \cdot L_d \cdot \frac{-ji_d}{\sqrt{3}} \cdot e^{jJ} - jw \cdot L_{hd} \cdot \frac{-ji_F^{'}}{\sqrt{3}} \cdot e^{jJ} \qquad (4.31)$$

We define:

$$\underline{I}_q = \frac{i_q}{\sqrt{3}} \cdot e^{jJ} \tag{4.32}$$

$$\underline{I}_d = \frac{-ji_d}{\sqrt{3}} \cdot e^{jJ} \tag{4.33}$$

$$\underline{I}'_F = \frac{-ji'_F}{\sqrt{3}} \cdot e^{jJ} \tag{4.34}$$

Then we obtain:

$$\underline{U}_d = -jw \cdot L_q \cdot \underline{I}_q \tag{4.35}$$

$$\underline{U}_q = -jw \cdot L_d \cdot \underline{I}_d - jw \cdot L_{hd} \cdot \underline{I}'_F \tag{4.36}$$

With appliance of reactances:

$$X_d = w \cdot L_d\,, \qquad X_q = w \cdot L_q\,, \qquad X_{hd} = w \cdot L_{hd} \tag{4.37 a-c}$$

and the synchronous generated voltage:

$$\underline{U}_P = -jX_{hd} \cdot \underline{I}'_F \tag{4.38}$$

follows:

$$\underline{U} = \underline{U}_d + \underline{U}_q = -jX_q \cdot \underline{I}_q - jX_d \cdot \underline{I}_d + \underline{U}_p \tag{4.39}$$

and respectively:

$$\underline{U}_p = \underline{U} + jX_q \cdot \underline{I}_q + jX_d \cdot \underline{I}_d \tag{4.40}$$

For this reason the phasor diagram of salient-pole machines can be drawn, if $\underline{U}$ and $\underline{I}$ and the directions of the d-(direct) and q-(quadrature) axes are known respectively predetermined.

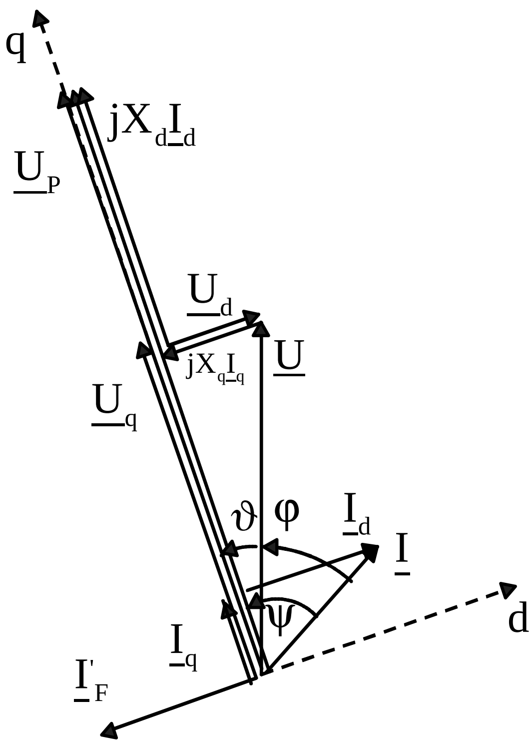


Fig. 56: SYM, phasor diagram

If decomposed in separate components follows:

$$\underline{U} = \underline{U}_d + \underline{U}_q \tag{4.41}$$

$$\underline{I} = \underline{I}_d + \underline{I}_q \tag{4.42}$$

$$y = J + j \tag{4.43}$$

$$I_d = I \cdot \sin y \ , \qquad I_q = I \cdot \cos y \tag{4.44 a,b}$$

$$U_d = X_q \cdot I_q = U \cdot \sin J \ , \qquad U_q = U_p - X_d \cdot I_d = U \cdot \cos J \tag{4.45 a,b}$$

For practical use mostly the following depiction of the phasor diagram is applied:

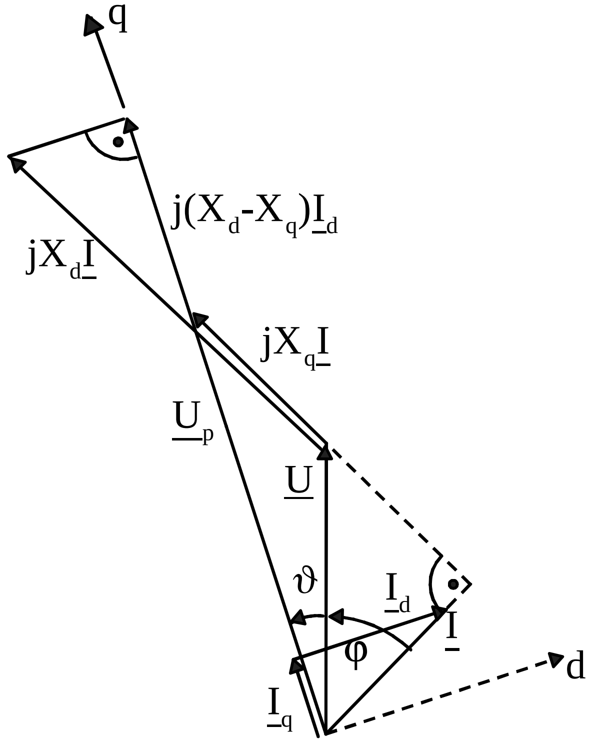


Fig. 57: SYM, common phasor diagram

<u>Construction manual:</u>

- from the power supply the type of load is predetermined: U , I , j are known
  $\rightarrow$ orientation of axes d and q is unknown
- phasors of $jX_q\underline{I}$ and $jX_d\underline{I}$ are to be drawn
- the q-axis, i.e. the direction of $\underline{U}_p$ and J , is determined by the straight line from the origin through the pivot of $jX_q\underline{I}$
- based on that, the current I can be decomposed into I_d and I_q
- the perpendicular from the pivot of $jX_d\underline{I}$ on the q-axis is due to the magnitude of $\underline{U}_p$

The phasor diagram of salient pole machines differs from that of cylindrical-rotor machines in the difference $\left(X_d - X_q\right)$, i.e. the different admittances in d- and q-axis $X_q \approx 0{,}5...0{,}8 \cdot X_d$. For $X_q = X_d = X$ the phasor diagram of the cylindrical-rotor machine is obtained.

If the direct components are substituted by the root-mean-square values, then the torque ensues to:

$$M_A = \frac{3p}{w} \cdot \left(\frac{u_d}{\sqrt{3}} \cdot \frac{i_d}{\sqrt{3}} + \frac{u_q}{\sqrt{3}} \cdot \frac{i_q}{\sqrt{3}} \right) \tag{4.46}$$

$$M_A = \frac{3p}{w} \cdot \left(U_d \cdot I_d + U_q \cdot I_q \right) \tag{4.47}$$

That means, that the active power in the d- and the q-axis are adding up.
With the relations found in the phasor diagram

$$I_d = \frac{U_p - U \cdot \cos J}{X_d}, \tag{4.48}$$

$$U_d = U \cdot \sin J \tag{4.49}$$

$$I_q = \frac{U \cdot \sin J}{X_q}, \tag{4.50}$$

$$U_q = U \cdot \cos J \tag{4.51}$$

the torque is converted and described as a function of the angular displacement.

$$M_A = \frac{3p}{w} \cdot \left[U \cdot \sin J \cdot \frac{U_p - U \cdot \cos J}{X_d} + U \cdot \cos J \cdot \frac{U \cdot \sin J}{X_q} \right] \tag{4.52}$$

$$= \frac{3p}{w} \cdot \left[\underbrace{\frac{U \cdot U_P}{X_d} \cdot \sin J}_{M_{Syn}} + \underbrace{\frac{U^2}{2} \cdot \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \cdot \sin 2J}_{M_{Rel}} \right] \tag{4.53}$$

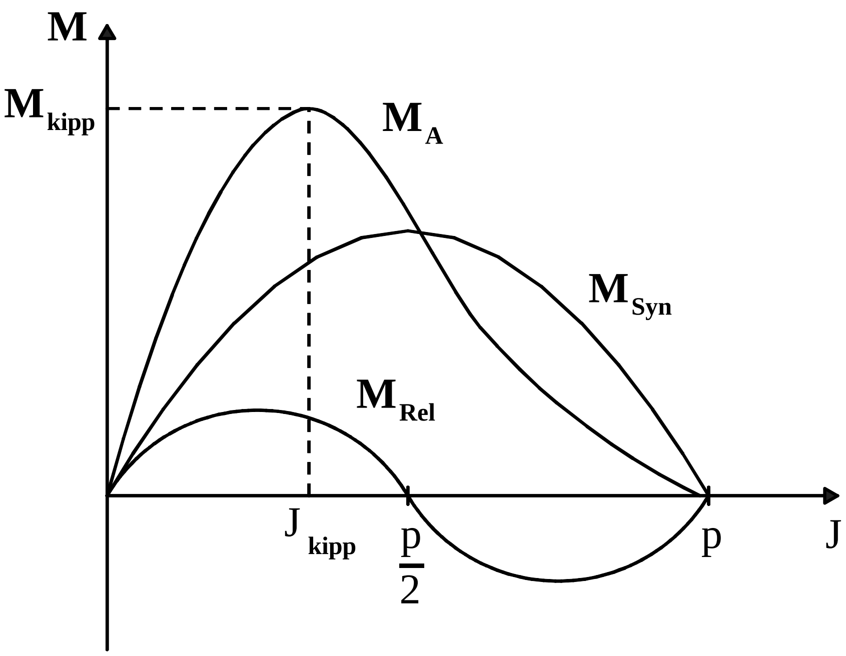


Fig. 58: SYM, torque vs. lagging angle J

The torque of salient-pole machines is composed of two components. The first part also appears in cylindrical-rotor machines and is depending on the excitation. It is called *synchronous torque*. The second part does not depend on the excitation and results from the difference of the permeances $X_d \neq X_q$. This is called the *reluctance torque*.

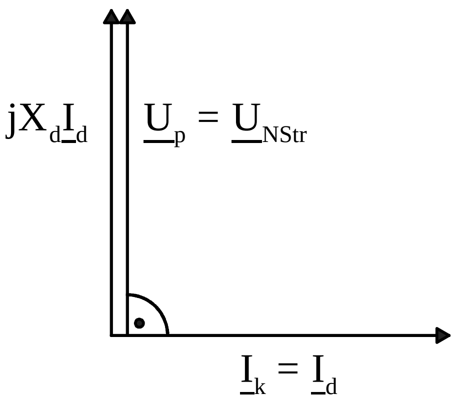
The magnetic unsymmetrical rotor tries to adjust itself in such a manner, that the magnetic energy is minimized. This does not depend on the polarity of the excitation. Therefore the reluctance torque has twice the frequency of the synchronous torque. The breakdown torque is moved to values $J_{Kipp} < \frac{p}{2}$.

In low power ranges machines without excitation winding are built, only using the reluctance torque caused by the difference of the permeances in d- and q-axis. These are the so called reluctance machines.

For $X_d = X_q = X$ the torque equation of the cylindrical-rotor machine is matched.

4.3 Determination of X_d and X_q

X_d can be determined with a measurement in continued short circuit with no-load excitation and pure direct axis field.



$$U = 0 \tag{4.54}$$

$$U_d = X_q \cdot I_q = 0 \rightarrow I_q = 0 \tag{4.55}$$

$$U_q = U_p - X_d \cdot I_d = 0 \tag{4.56}$$

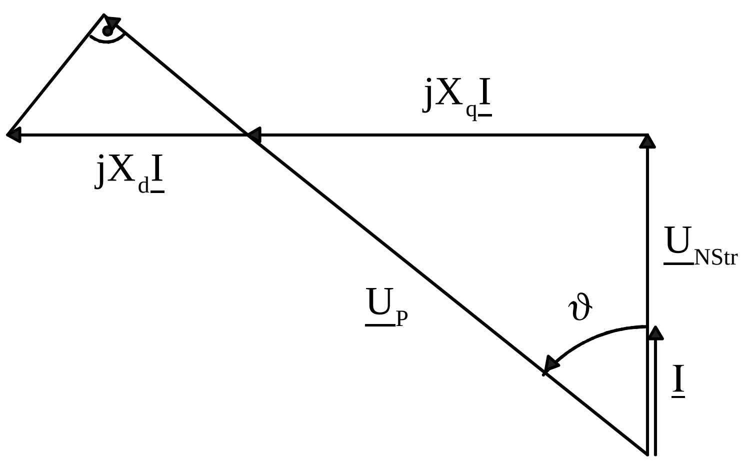
$$\rightarrow I_d = \frac{U_p}{X_d} = I_K \tag{4.57}$$

$$I_F = I_{F0} \; : \; U_p = U_{NStr} \tag{4.58}$$

$$X_d = \frac{U_{NStr}}{I_K} \tag{4.59}$$

Fig. 59: SYM, phasor diagram for continued short circuit

X_q can be determined with a measurement of the angular displacement when the machine is loaded with pure active-power load.



$$\cos j = 1 \tag{4.60}$$

$$\tan J = \frac{X_q \cdot I}{U_{NStr}} \tag{4.61}$$

$$X_q = \frac{U_{NStr}}{I} \cdot \tan J \tag{4.62}$$

Fig. 60: SYM; phasor diagram for pure active power load

Small salient-pole machines can be driven unexcited with a very small slip on the power supply. Depending on the rotor position concerning the stator field different currents appear depending on the width of air gap. The slip has to be very small (a few per mill), so that a damper winding has no influence on the measurement. Current and voltage, oscillating with the slip frequency, are measured.

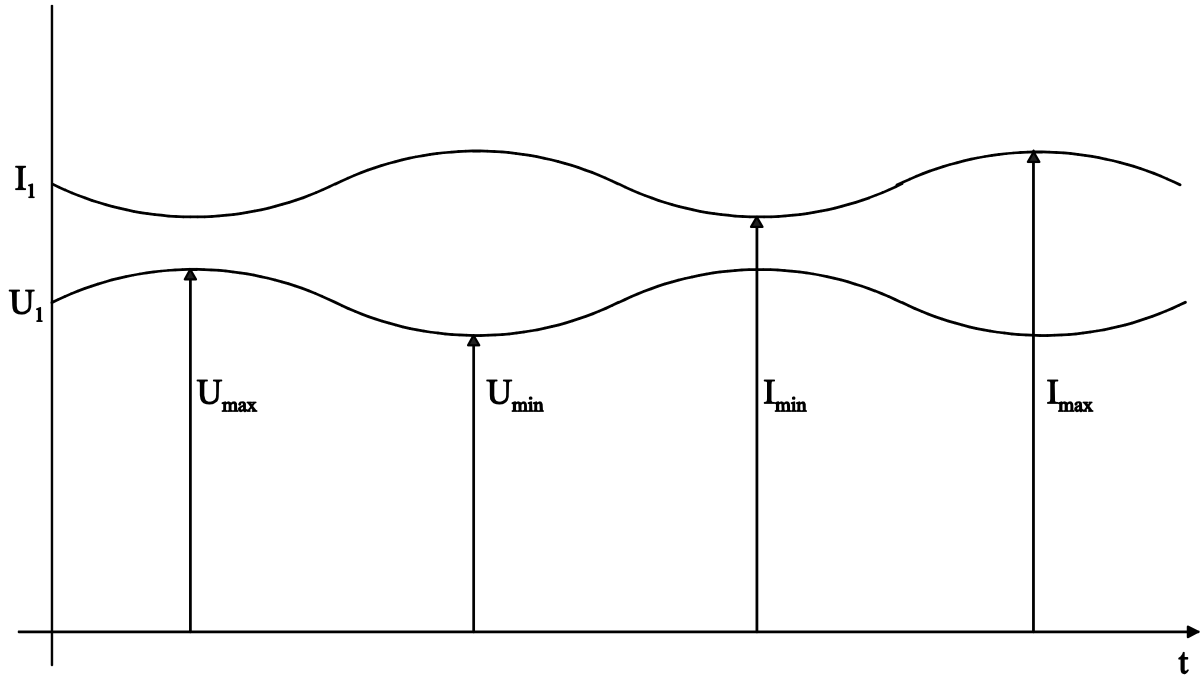


Fig. 61: SYM, current and voltage

Direct- X_d and quadrature component X_q of the so called synchronous reactance ensue to:

$$X_d = \frac{U_{max}}{I_{min}} \tag{4.63}$$

$$X_q = \frac{U_{min}}{I_{max}} \tag{4.64}$$

4.4 Sudden short circuit of the cylindrical-rotor machine

As an example for the use of the dynamic equations of the direct- and quadrature-axis theory for the calculation of dynamic phenomena, we now discuss the three-phase sudden short circuit of the cylindrical-rotor machine from the no-load operation.

The sudden short circuit is the transient phenomenon, which appears instantaneously after short-circuiting of the stator-circuit terminal. After all phenomena faded away, the machine is in continued (or sustained) short circuit.

The solution of this problem will be carried out analytically without using a computer. Therefore some simplifications have to be made for easing purposes:

- resistances of the stator winding are neglected, $R_1 = 0$.
- rotor speed remains constant during the dynamic phenomenon and is equal to the synchronous speed.
- The cylindrical-rotor machine is symmetrical $L_{hd} = L_{hq} = L_h$ with two identical rotor windings with an electrical displacement of 90°. The excitation winding with slip-rings is supplied with DC current. The quadrature-axis damper-winding is short-circuited. The machine has no direct-axis damper-winding.
- a rotating coordinate system is chosen with $\frac{da}{dt} = w = \frac{dg}{dt}$. The stator system has the denominators d and q with R_1 and L_1. The rotor system has the excitation winding F', the quadrature-axis damper-winding Q' with $R_2^{'}$ and $L_2^{'}$.
- reference axis is again the rotor direct axis. The initial condition is given by the switching instant $a = w \cdot t - \frac{p}{2} + e$.

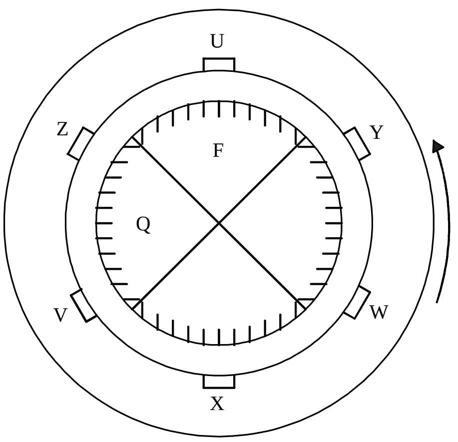


Fig. 62: SYM, orientation of windings

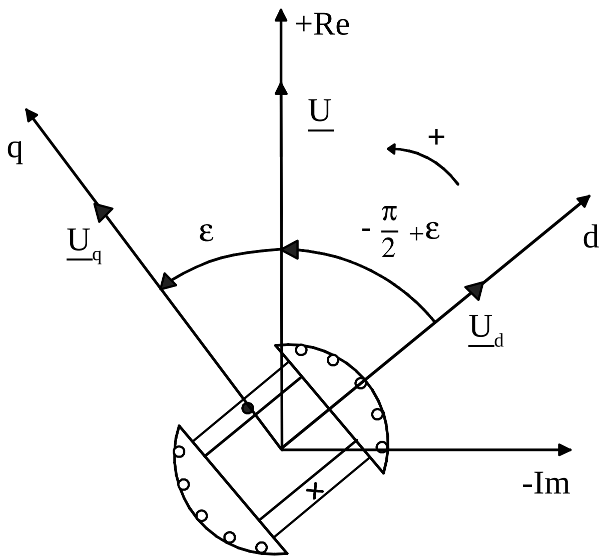


Fig. 63: SYM, revolving coordinate system

With the upper called simplifications the following set of equations is derived:

$$-u_d = \frac{d\Psi_d}{dt} - w \cdot \Psi_q \tag{4.65}$$

$$-u_q = \frac{d\Psi_q}{dt} + w \cdot \Psi_d \tag{4.66}$$

$$u_F^{'} = R_2^{'} \cdot i_F^{'} + \frac{d\Psi_F^{'}}{dt} \tag{4.67}$$

$$0 = R_2^{'} \cdot i_Q^{'} + \frac{d\Psi_Q^{'}}{dt} \tag{4.68}$$

$$M_A = p \cdot \left(\Psi_q \cdot i_d - \Psi_d \cdot i_q\right) \tag{4.69}$$

$$\Psi_d = L_1 \cdot i_d + L_h \cdot i_F^{'} \tag{4.70}$$

$$\Psi_q = L_1 \cdot i_q + L_h \cdot i_Q^{'} \tag{4.71}$$

$$\Psi_F^{'} = L_2^{'} \cdot i_F^{'} + L_h \cdot i_d \tag{4.72}$$

$$\Psi_Q^{'} = L_2^{'} \cdot i_Q^{'} + L_h \cdot i_q \tag{4.73}$$

The torque equation is decoupled and can be handled separately. Before starting the solution, initial conditions need to be pre-defined.

The initial state before switching is no-load operation and rated voltage:

$$i_d(0) = 0\,, \quad i_q(0) = 0\,, \quad i_Q^{'}(0) = 0\,, \quad i_F^{'}(0) = \sqrt{3} \cdot I_{F0}^{'} = \frac{u_F^{'}}{R_2^{'}} \tag{4.74 a-d}$$

$I_{F0}^{'}$ is the root-mean-square value of the no-load excitation current based on the stator, which causes the induction of rated voltage at rated speed. For this case the flux-linkages for the no-load operation are:

$$\Psi_d(0) = L_h \cdot i_F^{'}(0) \tag{4.75}$$

$$\Psi_q(0) = 0 \tag{4.76}$$

$$\Psi_F^{'}(0) = L_2^{'} \cdot i_F^{'}(0) \tag{4.77}$$

$$\Psi_Q^{'}(0) = 0\,. \tag{4.78}$$

Out of this follows the stator voltage for the no-load operation:

$$u_d(0) = 0 \tag{4.79}$$

$$u_q(0) = -w \cdot \Psi_d(0) = -w \cdot L_h \cdot i_F^{'}(0) = -X_h \cdot \sqrt{3} \cdot I_{F0}^{'} = -U_{NStr} \cdot \sqrt{3} \tag{4.80}$$

The inverse transformation with $a_0 = -\frac{p}{2} + e$ results in:

$$\underline{U} = \frac{u_d}{\sqrt{3}} \cdot e^{ja_0} + j\frac{u_q}{\sqrt{3}} \cdot e^{ja_0} = 0 + j\frac{-w \cdot L_h \cdot i_F^{'}(0)}{\sqrt{3}} \cdot e^{-j\frac{p}{2}} \cdot e^{je}$$

$$= -jX_h \cdot \frac{-ji_F^{'}(0)}{\sqrt{3}} \cdot e^{je} = -jX_h \cdot \underline{I}_{F0}^{'} = \underline{U}_{p0} = U_{NStr} \cdot e^{je} \tag{4.81}$$

with e being the switching angle

- if $e = 0$, then at the time $t = 0$ the voltage in phase U is in the real axis, i.e. switching at maximum voltage (flux-linkage equal zero).
- if $e = \pm\frac{p}{2}$, then at the time $t = 0$ the voltage in phase U is in the plus/minus imaginary axis, i.e. switching at voltage zero crossing (flux-linkage is maximum).

After defining the initial conditions the solution of the system of equations for $t \geq 0$ can be started. First the voltage equations of the stator are considered. For a three-phase short circuit follows:

$$-u_d = 0 = \frac{d\Psi_d}{dt} - w \cdot \Psi_q \tag{4.82}$$

$$-u_q = 0 = \frac{d\Psi_q}{dt} + w \cdot \Psi_d \tag{4.83}$$

The solution of these two differential equations can be easily done.
Ψ_q and Ψ_d are harmonic oscillations:

$$\Psi_q = A \cdot \cos wt + B \cdot \sin wt \tag{4.84}$$

$$\Psi_d = -B \cdot \cos wt + A \cdot \sin wt \tag{4.85}$$

The integration constants are determined by pasting the initial conditions:

$$\Psi_q(0) = A = 0 \tag{4.86}$$

$$\Psi_d(0) = -B = L_h \cdot i_F'(0) \tag{4.87}$$

For this case the according stator flux-linkages are obtained:

$$\Psi_d = L_h \cdot i_F'(0) \cdot \cos wt \tag{4.88}$$

$$\Psi_q = -L_h \cdot i_F'(0) \cdot \sin wt \tag{4.89}$$

Stator currents in the flux-linkage equations are eliminated by solving the stator flux-linkages for stator currents and pasting them into the rotor flux-linkages:

$$i_d = \frac{\Psi_d}{L_1} - \frac{i_F'}{1+s_1} \tag{4.90}$$

$$i_q = \frac{\Psi_q}{L_1} - \frac{i_Q'}{1+s_1} \tag{4.91}$$

$$\Psi_F' = \frac{1}{1+s_1} \cdot \left(\Psi_d + \frac{s}{1-s} \cdot L_h \cdot i_F' \right) = \frac{1}{1+s_1} \cdot \left(L_h \cdot i_F'(0) \cdot \cos wt + \frac{s}{1-s} \cdot L_h \cdot i_F' \right) \tag{4.92}$$

$$\Psi_Q' = \frac{1}{1+s_1} \cdot \left(\Psi_q + \frac{s}{1-s} \cdot L_h \cdot i_Q' \right) = \frac{1}{1+s_1} \cdot \left(-L_h \cdot i_F'(0) \cdot \sin wt + \frac{s}{1-s} \cdot L_h \cdot i_Q' \right) \tag{4.93}$$

Besides the known stator flux-linkages, the rotor flux-linkages still include the unknown rotor currents. By pasting the rotor flux-linkages into the rotor voltage equations the differential equations for the rotor currents are achieved:

$$u_F' = R_2' \cdot i_F'(0) = R_2' \cdot i_F' + \frac{1}{1+s_1} \cdot \left(-w \cdot L_h \cdot i_F'(0) \cdot \sin wt + \frac{s}{1-s} \cdot L_h \cdot \frac{di_F'}{dt} \right) \tag{4.94}$$

$$0 = R_2' \cdot i_Q' + \frac{1}{1+s_1} \cdot \left(-w \cdot L_h \cdot i_F'(0) \cdot \cos wt + \frac{s}{1-s} \cdot L_h \cdot \frac{di_Q'}{dt} \right) \tag{4.95}$$

It is to be divided by $R_2^{'}$ and needs to be defined ($\Rightarrow$ compare to transformers)

- open-circuit time constant of the rotor winding, open-coil stator:

$$T_{F0} = \frac{L_2^{'}}{R_2^{'}} = \frac{(1+s_2)\cdot L_h}{R_2^{'}} \tag{4.96}$$

- short-circuit time constant of the rotor winding, short-circuited stator:

$$T_{FK} = s \cdot T_{F0} \tag{4.97}$$

After conversion follows:

$$T_{FK} \cdot \frac{di_F^{'}}{dt} + i_F^{'} = i_F^{'}(0)\cdot\left(1 + w \cdot T_{FK} \cdot \frac{1-s}{s}\cdot \sin wt\right) \tag{4.98}$$

$$T_{FK} \cdot \frac{di_Q^{'}}{dt} + i_Q^{'} = i_F(0)\cdot w \cdot T_{FK} \cdot \frac{1-s}{s}\cdot \cos wt \tag{4.99}$$

These two inhomogeneous differential equations can be solved using Laplace transformation. Considering that $w \cdot T_{FK} >> 1$, i.e. $\frac{1}{w \cdot T_{FK}} \to 0$, solutions for the rotor currents follow as:

$$i_F^{'} = i_F^{'}(0)\cdot\left(1 - \frac{1-s}{s}\cdot \cos wt + \frac{1-s}{s}\cdot e^{-\frac{t}{T_{FK}}}\right) \tag{4.100}$$

$$i_Q^{'} = i_F^{'}(0)\cdot \frac{1-s}{s}\cdot \sin wt \tag{4.101}$$

An inverse transformation is not required, because the excitation winding and the damper winding are both arranged perpendicular and mounted on the rotating rotor. Now we know the stator flux-linkages and the rotor currents.
Rotor currents in the transformed system can be determined. Pasting known relations leads to:

$$i_d = \frac{L_h \cdot i_F^{'}(0)\cdot \cos wt}{L_1} - \frac{i_F^{'}(0)\cdot\left(1 - \frac{1-s}{s}\cdot \cos wt + \frac{1-s}{s}\cdot e^{-\frac{t}{T_{FK}}}\right)}{1+s_1} \tag{4.102}$$

$$i_q = \frac{-L_h \cdot i_F^{'}(0)\cdot \sin wt}{L_1} - \frac{i_F^{'}(0)\cdot \frac{1-s}{s}\cdot \sin wt}{1+s_1} \tag{4.103}$$

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \frac{i_F^{'}(0)}{1+s_1}\cdot \begin{bmatrix} \frac{1}{s}\cdot \cos wt - \left(1 + \frac{1-s}{s}\cdot e^{-\frac{t}{T_{FK}}}\right) \\ -\frac{1}{s}\cdot \sin wt \end{bmatrix} \tag{4.104}$$

Now the inverse transformation to the stationary three-phase system can be executed. Focus is put on the current in phase U:

$$i_u = \sqrt{\frac{2}{3}} \cdot i_A = \sqrt{\frac{2}{3}} \cdot \left(i_d \cdot \cos\left(wt - \frac{p}{2} + e \right) - i_q \cdot \sin\left(wt - \frac{p}{2} + e \right) \right) \tag{4.105}$$

After pasting, converting and using $i_F^{'}(0) = \frac{\sqrt{3} \cdot U_{NStr}}{w \cdot L_h}$ the stator current ensues to:

$$i_u = \frac{\sqrt{2} \cdot U_{NStr}}{w \cdot L_1} \cdot \left(-\left(1 + \frac{1-s}{s} \cdot e^{-\frac{t}{T_{FK}}} \right) \cdot \sin(wt + e) + \frac{1}{s} \cdot \sin e \right) \tag{4.106}$$

The stator current i_u now consists of an alternating component, which decreases from high to lower values, and a DC component, which does not fall off.
The bahavior of the DC component is physically not correct. The reason for this purpose is the neglect of the stator resistance, to make the system of equations solvable analytically. To obtain a universal valid solution for the time characteristic of the stator current, the declination of the DC component has to be added by the factor $e^{-t/T_{dK}}$.
T_{dK} is the short-circuit time constant of the stator winding.

$$T_{d0} = \frac{L_1}{R_1} \tag{4.107}$$

$$T_{dK} = s \cdot T_{d0} \tag{4.108}$$

Then follows:

$$i_u = \frac{\sqrt{2} \cdot U_{NStr}}{w \cdot L_1} \cdot \left(-\left(1 + \frac{1-s}{s} \cdot e^{-t/T_{FK}} \right) \cdot \sin(wt + e) + \frac{1}{s} \cdot \sin e \cdot e^{-t/T_{dK}} \right) \tag{4.109}$$

By introducing the synchronous reactance $X_1 = w \cdot L_1$ and the short-circuit reactance $X_{1K} = s \cdot X_1$ the stator current finally results in:

$$i_u = \sqrt{2} \cdot U_{NStr} \cdot \left\{ -\left[\frac{1}{X_1} + \left(\frac{1}{X_{1K}} - \frac{1}{X_1} \right) \cdot e^{-t/T_{FK}} \right] \cdot \sin(wt + e) + \frac{1}{X_{1K}} \cdot \sin e \cdot e^{-t/T_{dK}} \right\} \tag{4.110}$$

As well as for the DC component of the stator current, the declination has to be physically added in the corresponding alternating component of the rotor currents:

$$i_F^{'} = i_F^{'}(0) \cdot \left\{ 1 + \frac{1-s}{s} \cdot e^{-t/T_{FK}} - \frac{1-s}{s} \cdot \cos wt \cdot e^{-t/T_{dK}} \right\} \tag{4.111}$$

$$i_Q^{'} = i_F^{'}(0) \cdot \left\{ \frac{1-s}{s} \cdot \sin wt \cdot e^{-t/T_{dK}} \right\} \tag{4.112}$$

Now the results can be illustrated and interpreted:

- switching at maximum voltage: $e = 0$

$$u = \sqrt{2} \cdot U \cdot \cos wt \tag{4.113}$$

$$i_u = \sqrt{2} \cdot U_N \cdot \left[-\left\{ \frac{1}{X_1} + \left(\frac{1}{X_{1K}} - \frac{1}{X_1} \right) \cdot e^{-\frac{t}{T_{FK}}} \right\} \cdot \sin wt \right] \tag{4.114}$$

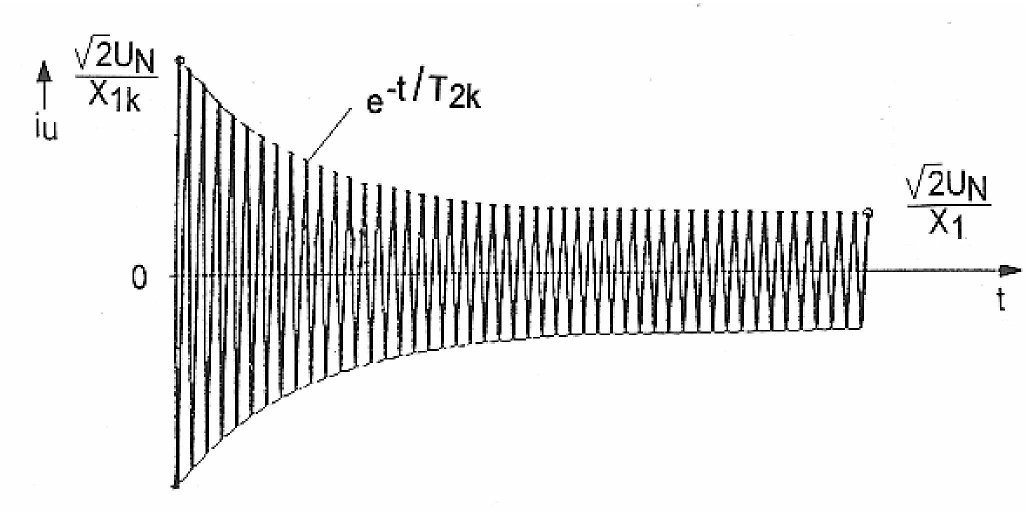


Fig. 64: voltage, current vs. time

- switching at voltage zero crossing: $e = \frac{p}{2}$

$$u = \sqrt{2} \cdot U \cdot \sin wt \tag{4.115}$$

$$i_u = \sqrt{2} \cdot U_N \cdot \left[-\left\{ \frac{1}{X_1} + \left(\frac{1}{X_{1K}} - \frac{1}{X_1} \right) \cdot e^{-\frac{t}{T_{FK}}} \right\} \cdot \cos wt + \frac{1}{X_{1K}} \cdot e^{-\frac{t}{T_{dK}}} \right] \tag{4.116}$$

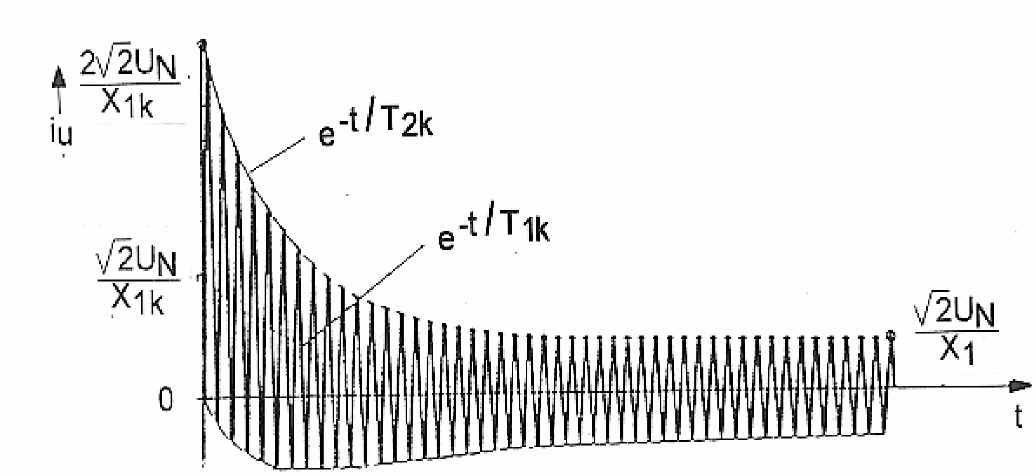


Fig. 65: voltage, current vs. time

$$i_F^{'} = i_F^{'}(0) \cdot \left[1 + \frac{1-s}{s} \cdot e^{-\frac{t}{T_{FK}}} - \frac{1-s}{s} \cdot \cos wt \cdot e^{-\frac{t}{T_{dK}}} \right] \tag{4.117}$$

Fig. 66: exciter current vs. time

$$i_Q^{'} = i_F^{'}(0) \cdot \frac{1-s}{s} \cdot \sin wt \cdot e^{-\frac{t}{T_{dK}}} \tag{4.118}$$

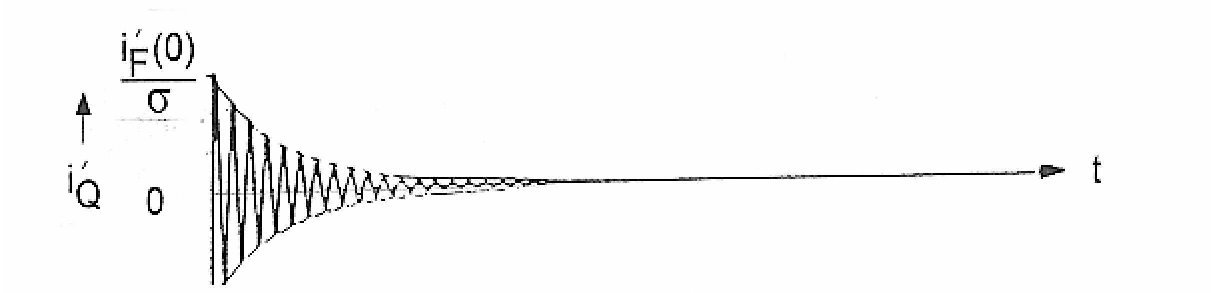


Fig. 67: damper current vs. time

- The time characteristic of the stator currents in the stator windings depends on the instant of switching, because of the sinusoidal changing flux-linkage before switching.
- Switching at maximum voltage: There is no DC component. The short-circuit current has a time-delay of $\frac{p}{2}$ and starts at zero crossing $\left(I_{u\max} = \frac{\sqrt{2} \cdot U_{NStr}}{X_{1K}} \right)$.
- Switching at voltage zero crossing: There is a DC component in full, because the short-circuit current with a time-delay of $\frac{p}{2}$ has to be compensated to zero. After one period the DC component and the alternating component add up to twice their value. The DC component declines with T_{dK} . $\left(I_{u\max} = 2 \cdot \frac{\sqrt{2} \cdot U_{NStr}}{X_{1K}} \right)$
- Instantaneously after switching the magnitude of the short-circuit current relevantly depends on the short-circuit reactance $X_{1K} = s \cdot X_1$. The initial symmetrical short-circuit current $\frac{\sqrt{2} \cdot U_{NStr}}{X_{1K}}$ flows. The initial symmetrical short-circuit current declines with the short-circuit time constant T_{FK} to the sustained short-circuit current $\frac{\sqrt{2} \cdot U_{NStr}}{X_1}$, which depends on the synchronous reactance.

- The time characteristic of the rotor currents does not depend on the instant of switching, because the flux-linkage before switching was constant.
- The DC component of the stator current corresponds to the alternating component of the rotor. The alternating component of the stator current corresponds to the DC component of the rotor.
- After switching the flux linkage can not change in the short-circuited stator winding. Therefore the currents in the excitation winding and in the damper winding have to magnetize oppositely and counteracting. Because of the resistance of the stator winding, the alternating component declines with T_{dK} . The DC component is increased as well as the stator current with a factor of $\frac{1}{s}$ and declines as well as the alternating component of the stator current with $\frac{1}{T_{FK}}$.
- In practice the following values are typical for cylindrical-rotor generator:
    - $x_1 = \frac{X_1}{U_N / I_N} = \frac{I_N}{I_{K0}} = 1{,}2 \cdots 2$
    - $x_{1K} = s \cdot x_1 = 0{,}15 \cdots 0{,}25$
    - $T_{dK} = 60 \cdots 250 ms$
    - $T_{FK} = 0{,}5 \cdots 2s$

4.4.1 Physical explication of the sudden short circuit

If synchronous machines are driven in no-load operation with rated voltage or if driven with rated load with sudden short-circuit of the terminals, short-circuit currents occur, which are a multiple of the sustained short-circuit current. Current peaks also occurring in the rotor are multiple times higher than the excitation current in operation at rated values.

Figures 68 and 69, showing a snapshot of the flux distribution in the synchronous machine before the sudden short circuit and half a period after, give a simple physical explication.

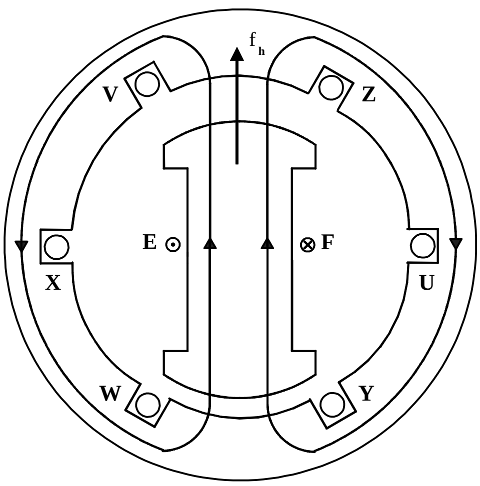


Fig. 68: $wt = 0$

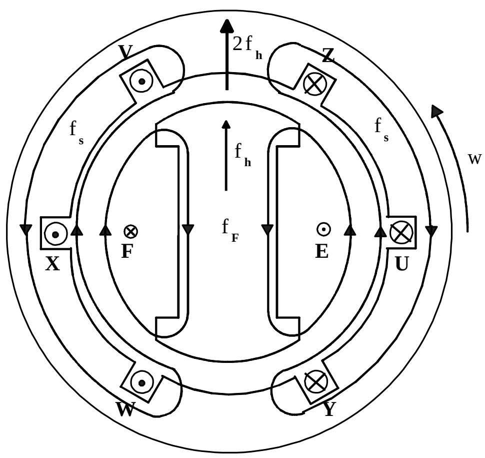


Fig. 69: $wt = p$

For $wt < 0$ the flux-linkage in the stator winding U is maximal. Therefore the voltage in phase U is:

$$u = \sqrt{2} \cdot U \cdot \sin wt \tag{4.119}$$

And the flux-linkages are:

$$\Psi_1 = L_h \cdot \sqrt{2} \cdot I_{Fo}^{'} \tag{4.120}$$

$$\Psi_2 = (1 + s_2) \cdot L_h \cdot \sqrt{2} \cdot I_{F0}^{'} = L_2^{'} \cdot \sqrt{2} \cdot I_{F0}^{'} \tag{4.121}$$

The synchronous machine is three-phase short circuited, when phase U has its voltage zero crossing. With neglect of the resistances of stator winding and rotor winding we obtain in stator and rotor (for $wt \geq 0$):

$$u = \frac{d\Psi}{dt} = 0 \ , \quad \text{i.e. } \Psi = const \tag{4.122}$$

i.e. a constant flux-linkage is forced. This means, that the flux f_h, whose magnetic loop was closed along the stator at no-load operation, keeps its magnitude but is now displaced to the magnetically worse conductive leakage path. If the rotor has turned half a revolution, the stator winding has to generate a current-linkage, which is able to drive a flux with the double magnitude of the pole-flux through the leakage path, to retain the original flux-linkage.

For $wt = p$ follows:

$$\Psi_1 = -L_h \cdot \sqrt{2} \cdot I_{F0}^{'} + (1 + s_1) \cdot s \cdot L_h \cdot \sqrt{2} \cdot I_1 \overset{!}{=} L_h \cdot \sqrt{2} \cdot I_{F0}^{'} \tag{4.123}$$

and then

$$\sqrt{2} \cdot I_1 = \frac{2\sqrt{2} \cdot I_{F0}^{'} \cdot L_h}{s \cdot L_1} = \frac{2\sqrt{2} \cdot U_N}{s \cdot X_1} \tag{4.124}$$

The sustained short-circuit current is increased with a factor of $\frac{1}{s}$. The relevant reactance is $X_{1K} = s \cdot X_1$. When switching at voltage zero crossing, the DC component occurs entirely, which doubles the amplitude of the sudden short-circuit current.

4.4.2 Torque at sudden short circuit

The dynamic calculation shows, that the occurring currents at sudden short circuit are much higher than the sustained short-circuit current. Consequentially the forces in the machine are much higher and have to be taken into consideration for the construction of synchronous machines. Therefore the torque at sudden short circuit is to be estimated. We want to consider the maximal mechanical stresses so the declination of the currents is neglected, leading to:

- $e^{-t/T_{FK}} = 1$
- $e^{-t/T_{dK}} = 1$

Then the torque and the currents ensue to:

$$M_{el} = p \cdot L_h \cdot \left(i_{q1} \cdot i_F^{'} - i_{d1} \cdot i_Q^{'} \right) \tag{4.125}$$

$$i_d = \frac{\sqrt{3} \cdot U_{NStr}}{w \cdot L_1} \cdot \left(\frac{1}{s} \cdot \cos wt - \left(1 + \frac{1-s}{s} \right) \right) = \frac{\sqrt{3} \cdot U_{NStr}}{s \cdot w \cdot L_1} \cdot \left(\cos wt - 1 \right) \tag{4.126}$$

$$i_q = \frac{\sqrt{3} \cdot U_{NStr}}{s \cdot w \cdot L_1} \cdot \left(- \sin wt \right) \tag{4.127}$$

$$i_F^{'} = \frac{\sqrt{3} \cdot U_{NStr}}{w \cdot L_h} \cdot \left(1 - \frac{1-s}{s} \cdot \cos wt + \frac{1-s}{s} \right) = \frac{\sqrt{3} \cdot U_{NStr}}{s \cdot w \cdot L_h} \cdot \left(1 - (1-s) \cdot \cos wt \right) \tag{4.128}$$

$$i_Q^{'} = \frac{\sqrt{3} \cdot U_{NStr}}{s \cdot w \cdot L_h} \cdot \left((1-s) \cdot \sin wt \right) \tag{4.129}$$

Pasting and converting leads to:

$$M_{elK} = \frac{-3p}{w} \cdot \frac{U_{NStr}^2}{s \cdot X_1} \cdot \sin wt \tag{4.130}$$

The peak transient torque oscillates at system frequency. Based on the rated torque results:

$$m_{el} = \frac{M_{elK}}{M_N} = \frac{-\frac{3p}{w} \cdot \frac{U_{NStr}^2}{s \cdot X_1} \cdot \sin wt}{\frac{3p}{w} \cdot U_N I_N \cos j_N} = \frac{-1}{x_K \cdot \cos j_N} \cdot \sin wt \tag{4.131}$$

If the numerical values are calculated, it can be seen, that the peak transient torque is increased by a factor

$$\frac{1}{0{,}15 \cdot 0{,}7} \cdots \frac{1}{0{,}25 \cdot 0{,}8} = 9{,}5 \cdots 5$$

related to the according rated torque. This is important for the dimensioning of the mechanical system. The torque results from the co-action of the DC component of the stator current and the rotating field of the rotor.

4.5 Sudden short circuit of salient-pole machines

4.5.1 Analytical calculation

Subject of discussion will be the sudden short circuit at no-load operation and rated voltage of the salient-pole machine again using simplifying assumptions. The complexity of the calculation is considerable a result can be given directly.
Stator voltage

$$u_u = \sqrt{2} \cdot U_N \cdot \sin(wt + e) \tag{4.132}$$

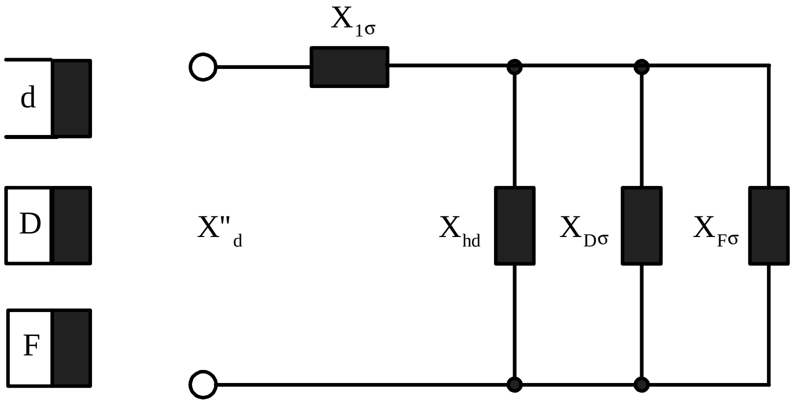
Stator current:

$$i_u = \sqrt{2} \cdot U_{N,Str} \cdot \left\{ \left[\underbrace{\left(\frac{1}{X_d^{"}} - \frac{1}{X_d^{'}} \right) \cdot e^{-\frac{t}{T_d^{"}}}}_{subtransient...} + \underbrace{\left(\frac{1}{X_d^{'}} - \frac{1}{X_d} \right) \cdot e^{-\frac{t}{T_d^{'}}}}_{transient...} + \underbrace{\frac{1}{X_d}}_{steady\ state...} \right] \cdot \cos(wt + e) \ ... \right.$$

$$\left. ... \ \underbrace{- \cos e \cdot \frac{1}{2} \cdot \left(\frac{1}{X_d^{"}} + \frac{1}{X_q^{"}} \right) \cdot e^{-\frac{t}{T_A}}}_{asymmetric...} \underbrace{- \cos(2wt + e) \cdot \frac{1}{2} \cdot \left(\frac{1}{X_d^{"}} - \frac{1}{X_q^{"}} \right) \cdot e^{-\frac{t}{T_A}}}_{double-frequent} \right\} \tag{4.133}$$

The single components of the short-circuit current are determined by the magnetizing reactances and the leakage reactances of the concerned windings. For the occurring couplings, the equivalent magnitudes $X_d^{"}$, $X_q^{"}$ and $X_d^{'}$ are defined.

The subtransient reactance $X_d^{"}$ is the effective reactance in the d-axis at first point of time. Here the stator winding d, the damper winding D and the excitation winding F feature a magnetic coupling at first. The reactance at sudden short circuit results from the magnetizing reactance and the leakage reactances ($\Rightarrow$ compare with transformers). The according transient time constant $T_d^{"}$ is obtained as well.



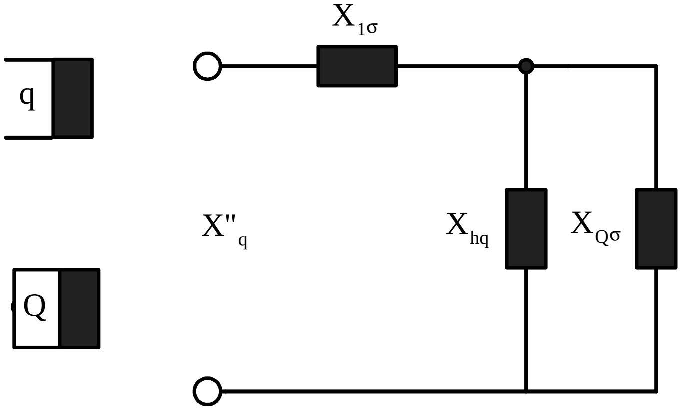
$$X_d^{"} = X_{1s} + X_{hd} \| X_{Ds} \| X_{Fs}$$

$$T_d^{"} = \frac{L_d^{"}}{R_1}$$

$$I_K^{"} = \frac{U_{NStr}}{X_d^{"}}$$

Fig. 70: exciter-, damper windings

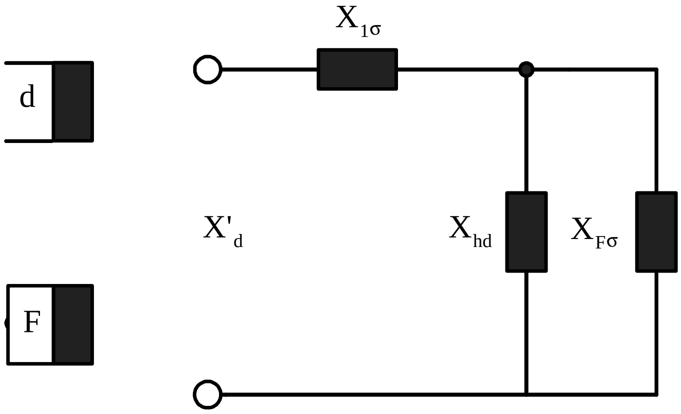
Besides the stator winding q only the quadrature-axis damper-winding Q is active in the q-axis. For the calculation of the quadrature-axis subtransient reactance $X_q^{''}$, the simple parallel connection of the magnetizing reactance and the damper winding in the q-axis are gained.



$$X_q^{''} = X_{1s} + X_{hq} \| X_{Qs}$$

Fig. 71: quadrature components

If the influence of the damper winding is declined rapidly, still the excitation winding and the short circuited stator winding in the d-axis are coupled. The transient reactance $X_d^{'}$ and the transient time constant $T_d^{'}$ are calculated as follows:



$$X_d^{'} = X_{1s} + X_{hd} \| X_{Fs}$$

$$T_d^{'} = \frac{L_d^{'}}{R_1}$$

$$I_K^{'} = \frac{U_{NStr}}{X_d^{'}}$$

Fig. 72: direct components

A transient reactance $X_q^{'}$ does not exist, because there is no other winding in the rotor q-axis besides the damper winding.
After the declination of the longer lasting transient process, the stator current fades to the sustained short-circuit current, which is determined by the direct reactance X_d.

$$I_K = \frac{U_{NStr}}{X_d} \tag{4.134}$$

The appearing DC component depends on the instant of switching. Switching at zero voltage crossing results in an entire DC component. If the switching is executed at maximum voltage, no DC component comes up. The DC component falls off with the asymmetric time constant T_A.

$$T_A = \frac{1}{R_1} \cdot \frac{2 \cdot L_q^{''} \cdot L_d^{''}}{L_q^{''} + L_d^{''}} \tag{4.135}$$

The double-frequency component, as well dropping with the asymmetric time constant T_A, results from the magnetic asymmetry $X_d^{''} \neq X_q^{''}$ at first instant.
Standard machines with direct-axis and quadrature-axis damper-winding are not asymmetrical because of $X_d^{''} \approx X_q^{''}$.
For this reason the double-frequency component disappears and the time constant T_A is equal to the subtransient time constant $T_A \approx T_d^{''}$.

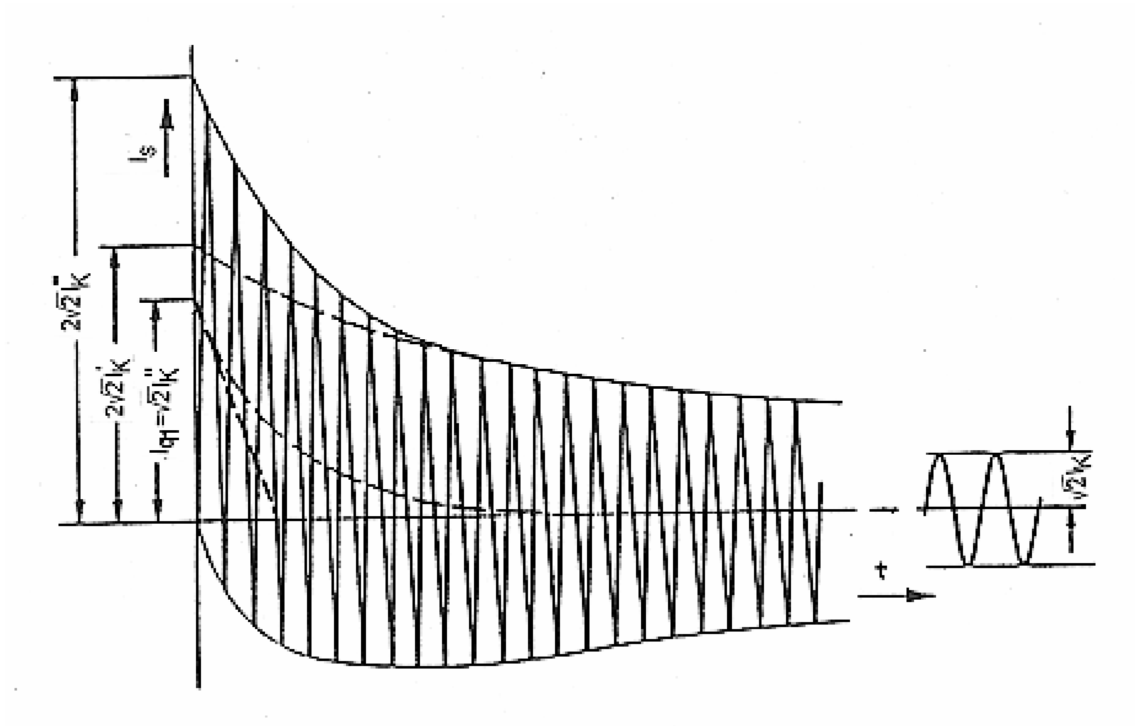


Fig. 73: sudden short-circuit current with maximum DC component, one phase

The maximum peak of the sudden short-circuit current in phase U occurs half a period after short-circuiting the three phases at voltage zero crossing of phase U. The amplitudes of the subtransient and the DC component add up. Regarding the damping, the current peak is:

$$I_S = \sqrt{2} \cdot 1{,}8 \cdot \frac{U_{NStr}}{X_d^{''}} \tag{4.136}$$

In agreement with a VDE (association of german electrical engineers), the maximum value has to be smaller than 15 times the peak value of the rated current, because of the high occurring forces and oscillating torques.

- $\frac{I_S}{\sqrt{2} \cdot I_N} = \frac{1{,}8 \cdot U_{NStr}}{X_d^{''} \cdot I_N} = \frac{1{,}8}{X_d^{''}} \leq 15$
- $X_d^{''} \geq \frac{1{,}8}{15} = 0{,}12$

Typical values of the reactances and time constants of salient-pole machines with definite damper winding are:

- $x_d = 0{,}8 \cdots 1{,}4\,\text{p.u.}$
- $x_q = 0{,}4 \cdots 0{,}9\,\text{p.u.}$
- $x_d^{'} = 0{,}2 \cdots 0{,}4\,\text{p.u.}$
- $x_d^{''} \approx x_q^{''} = 0{,}12 \cdots 0{,}25\,\text{p.u.}$
- $T_d^{'} = 0{,}5 \cdots 2{,}5s$
- $T_d^{''} \approx T_A = 0{,}02 \cdots 0{,}1s$

Those reactances and time constants can be determined from the sudden short-circuit.

4.5.2 Numerical solution

Using numerical integration methods on the computer, the complete system of equations of synchronous machines can be directly solved without any simplifying assumptions. Particularly the assumption of a constant speed is not to apply anymore.

Indeed the system of equations has to be converted into *state form*, which is well known from control engineering.

$$\frac{d\Psi_d}{dt} = -u_d - i_d \cdot R_1 + w \cdot \Psi_q \tag{4.137}$$

$$\frac{d\Psi_q}{dt} = -u_q - i_q \cdot R_1 - w \cdot \Psi_d \tag{4.138}$$

$$\frac{d\Psi_F^{'}}{dt} = u_F^{'} - i_F^{'} \cdot R_F^{'} \tag{4.139}$$

$$\frac{d\Psi_D^{'}}{dt} = -i_D^{'} \cdot R_D^{'} \tag{4.140}$$

$$\frac{d\Psi_Q^{'}}{dt} = -i_Q^{'} \cdot R_Q^{'} \tag{4.141}$$

$$\frac{dw}{dt} = \frac{p}{J} \cdot \left[M_A - p \cdot \left(\Psi_q \cdot i_d - \Psi_d \cdot i_q\right)\right] \tag{4.142}$$

Therefore the inductivity matrix has to be inverted. This is not explained in detail here.

$$\begin{bmatrix} i_d \\ i_F^{'} \\ i_D^{'} \\ i_q \\ i_Q^{'} \end{bmatrix} = [L]^{-1} \begin{bmatrix} \Psi_d \\ \Psi_F^{'} \\ \Psi_D^{'} \\ \Psi_q \\ \Psi_Q^{'} \end{bmatrix} \qquad [L] = \begin{bmatrix} L_d & L_{hd} & L_{hd} & 0 & 0 \\ L_{hd} & L_F^{'} & L_{hd} & 0 & 0 \\ L_{hd} & L_{hd} & L_D^{'} & 0 & 0 \\ 0 & 0 & 0 & L_q & L_{hq} \\ 0 & 0 & 0 & L_{hq} & L_Q^{'} \end{bmatrix} \tag{4.143}$$

For this reason the complete system of equations is known as interpreted in the shown structure diagram (Fig. 74), which has to be solved – using:

- excitation values: u_d, u_q, u_F, M_A
- state values: Ψ_d, Ψ_q, $\Psi_F^{'}$, $\Psi_D^{'}$, $\Psi_Q^{'}$, w

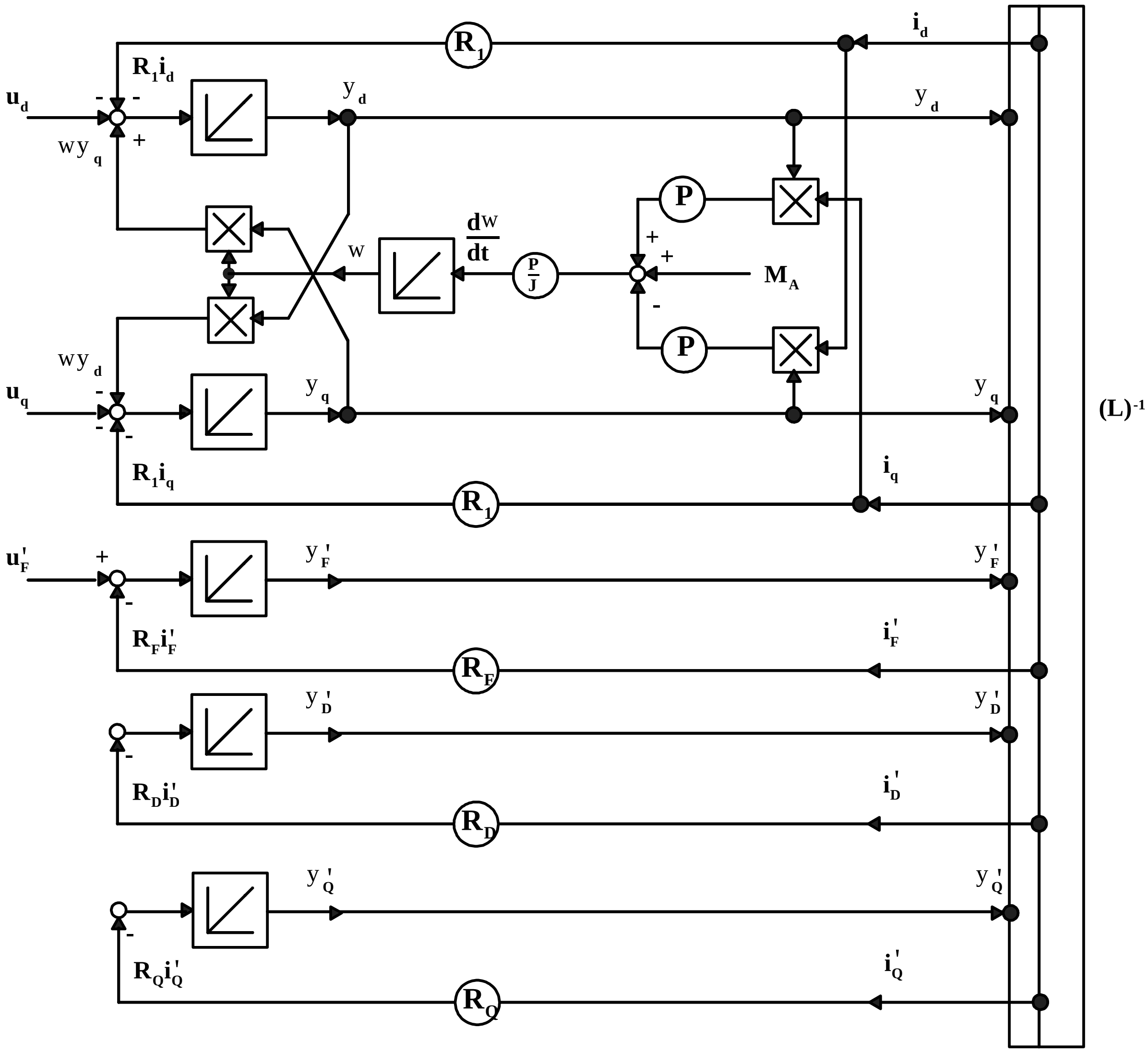


Fig. 74: structure diagram of salient-pole machines

1) Initial conditions for $t < 0$ need to be predefined, for example no-load operation at rated voltage and synchronous speed.

$$i_d(0) = i_q(0) = i_D^{'}(0) = i_Q^{'}(0) = 0 \quad (4.144)$$

$$u_d(0) = +w \cdot \Psi_q(0) = 0 \quad (4.145)$$

$$u_q(0) = -w \cdot \Psi_d(0) = -w \cdot L_{hd} \cdot i_F^{'}(0) = -\sqrt{3} \cdot U_{NStr} \quad (4.146)$$

$$u_F^{'}(0) = R_F^{'} \cdot i_F^{'}(0) \quad (4.147)$$

and $M_A = 0$.

Practically we define $a = w \cdot t - \frac{p}{2} + e$. Then it is:

$$u_{1u0} = \sqrt{\frac{2}{3}} \cdot \left(u_d(0) \cdot \cos a - u_q(0) \cdot \sin a\right) = \sqrt{\frac{2}{3}} \cdot \left(0 + \sqrt{3} \cdot U_{NStr} \cdot \sin\left(wt - \frac{p}{2} + e\right)\right)$$

$$= \sqrt{2} \cdot U \cdot \cos(wt + e) \tag{4.148}$$

- $e = 0$ switching at maximum voltage
- $e = \pm \frac{p}{2}$ switching at zero voltage crossing

2.) The following machine parameters need to be known:

L_h, c_d, c_q

R_1, R_F, $R_D = R_Q$

U_{1N}, U_F

J, M_N, f_N, p

and the leakage factors of the single windings:

s_1, s_F, $s_D = s_Q$

3.) Excitation values:

For $t > 0$ the stator voltages for the sudden short-circuit are substituted by zero.

$u_d = u_q = 0$

For coarse synchronizing the stator voltages are:

$u_d = 0$, $u_q = \sqrt{3} \cdot U_{NStr}$

The two other excitation states are:

$M_A = 0$, $u'_F = R_F \cdot i'_F(0)$

4.) For example a machine is chosen with the following data:

- apparent power: $S_N = 100kVA$
- nominal voltage: $U_N = 380V(verk.)$
- power factor: $\cos j_N = 0{,}8$
- pole pairs: $p = 3$
- frequency: $f_N = 50Hz$
- nominal current: $I_N = 152A$
- nominal torque: $M_N = 764Nm$

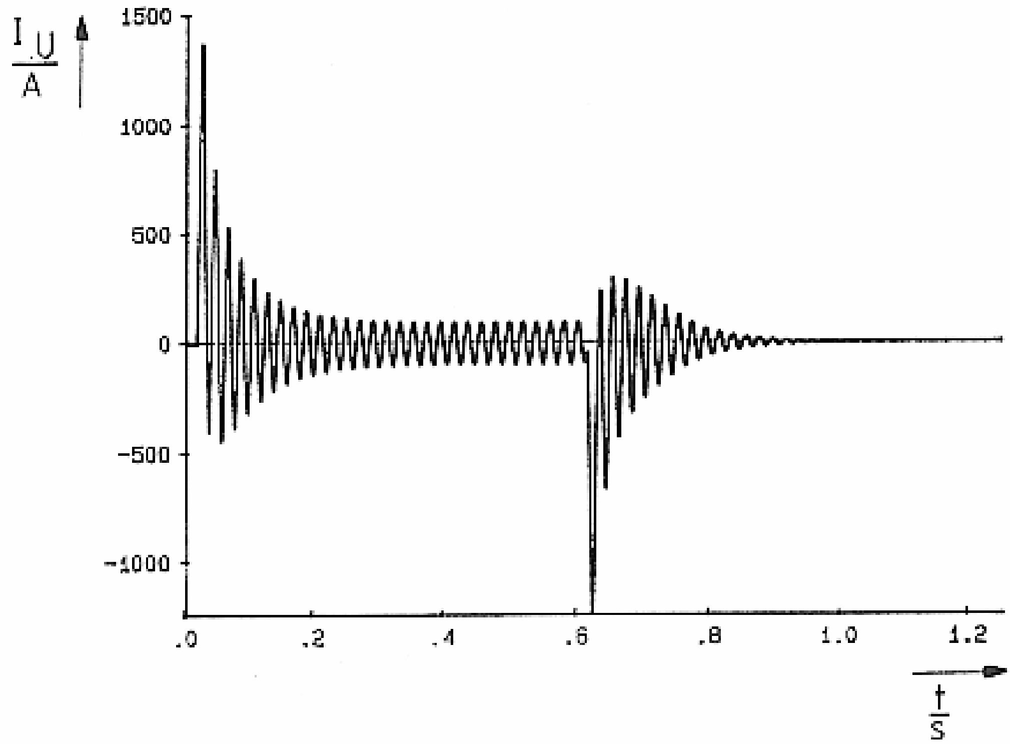


Fig. 75: salient-pole machine, sudden short-circuit and coarse synchronizing: stator current i_U

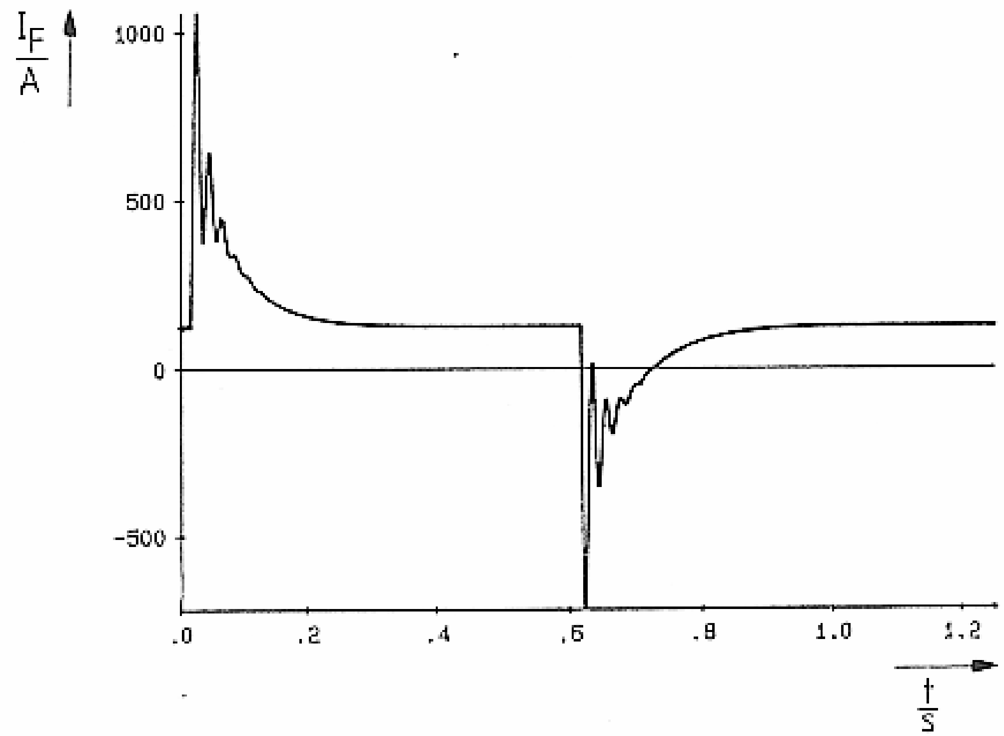


Fig. 76: sal. pole machine, sudden short-circuit and coarse synchronizing:excitation current i_F

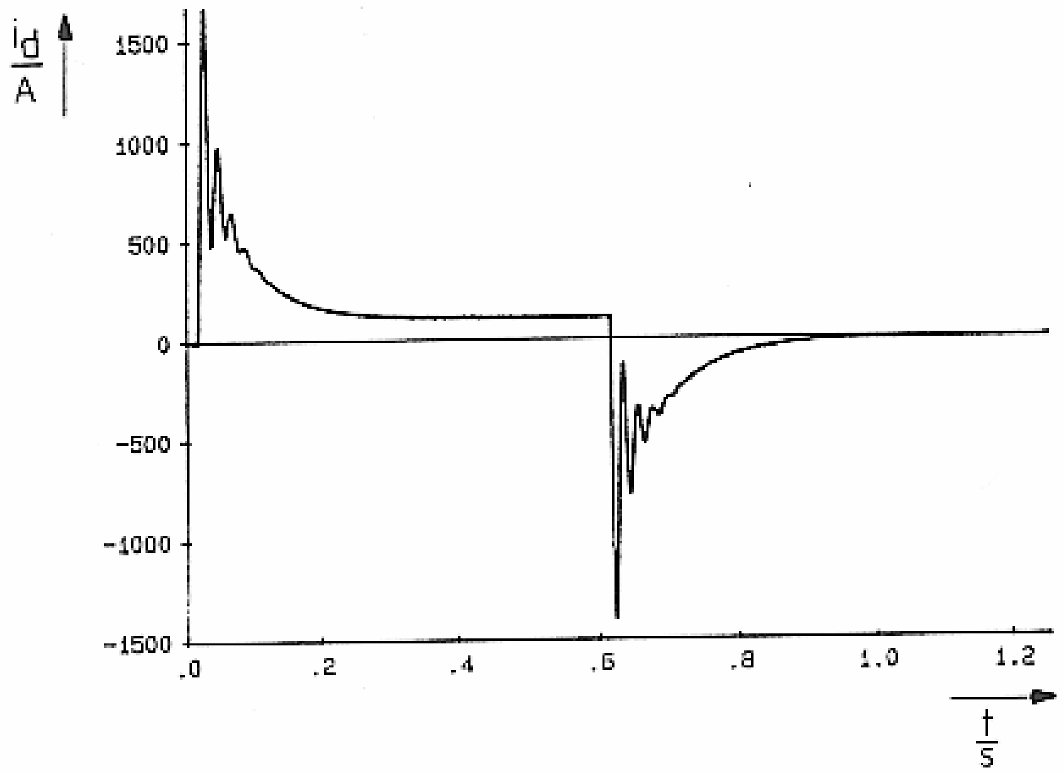


Fig. 77: sal. pole machine, sudden short-circuit and coarse synchr.: direct stator current i_d

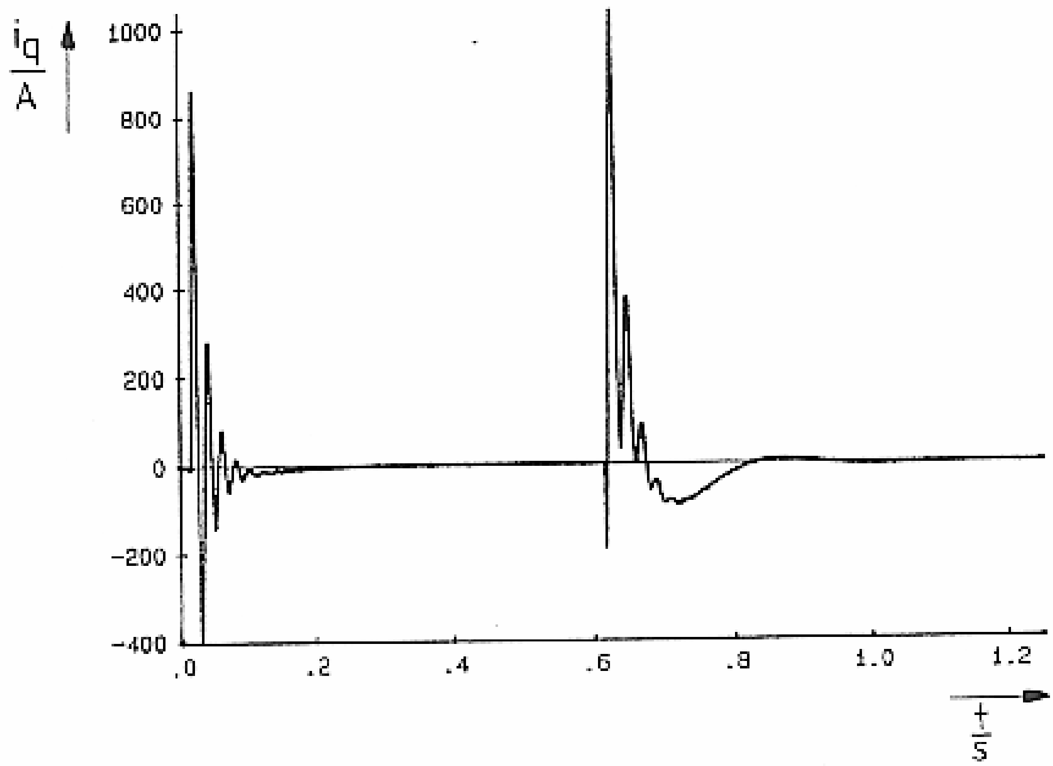


Fig. 78: sal. pole mach., sudden short-circuit and coarse synchr.: quadrature stator current i_q

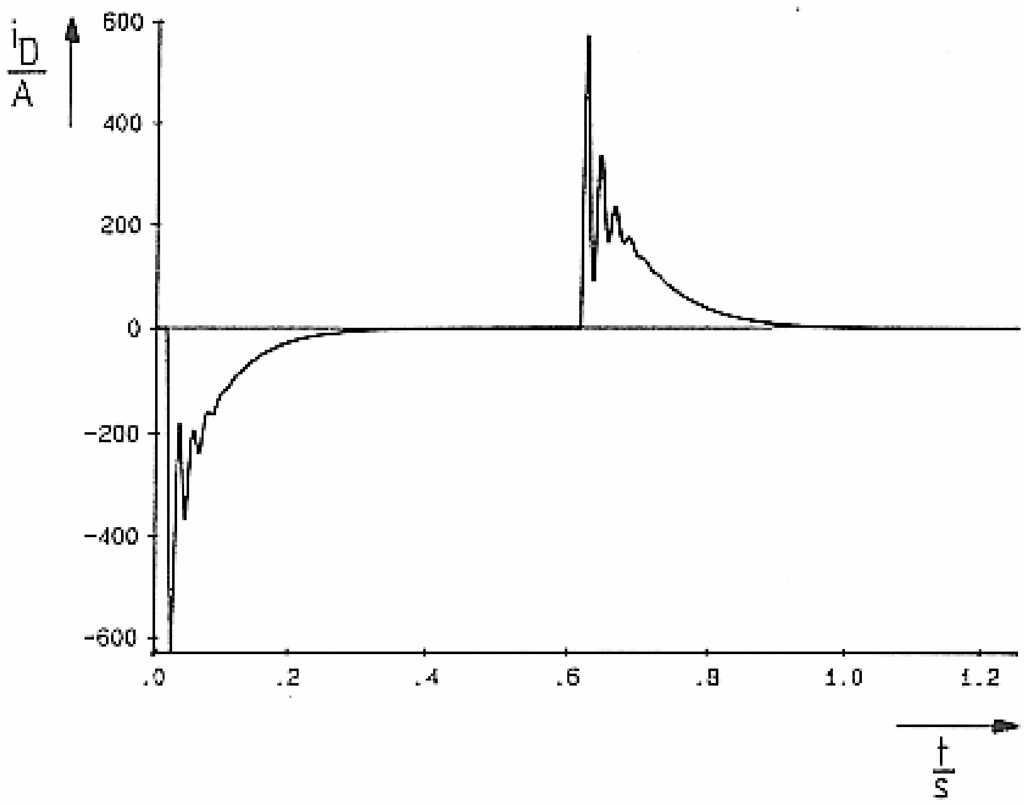


Fig. 79: salient pole mach., sudden short-circuit and coarse synchr.: direct damper current i_D

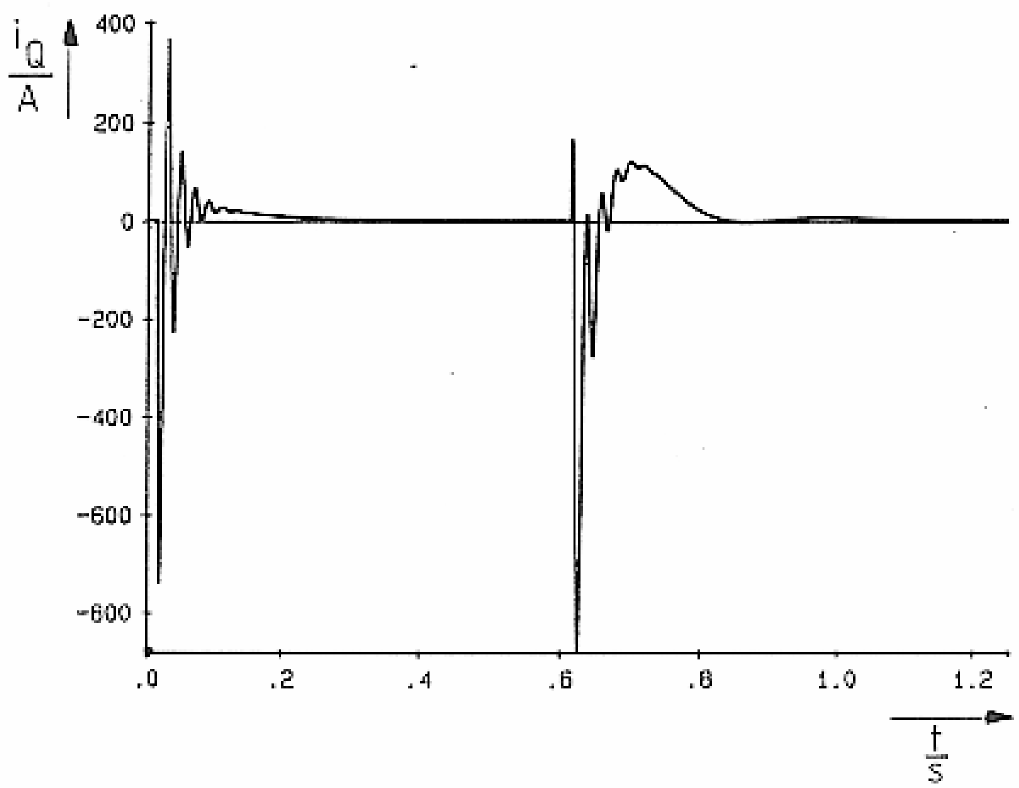


Fig. 80: sal. pole mach., sudden short-circuit and coarse synchr.: quadrature damp. current i_Q

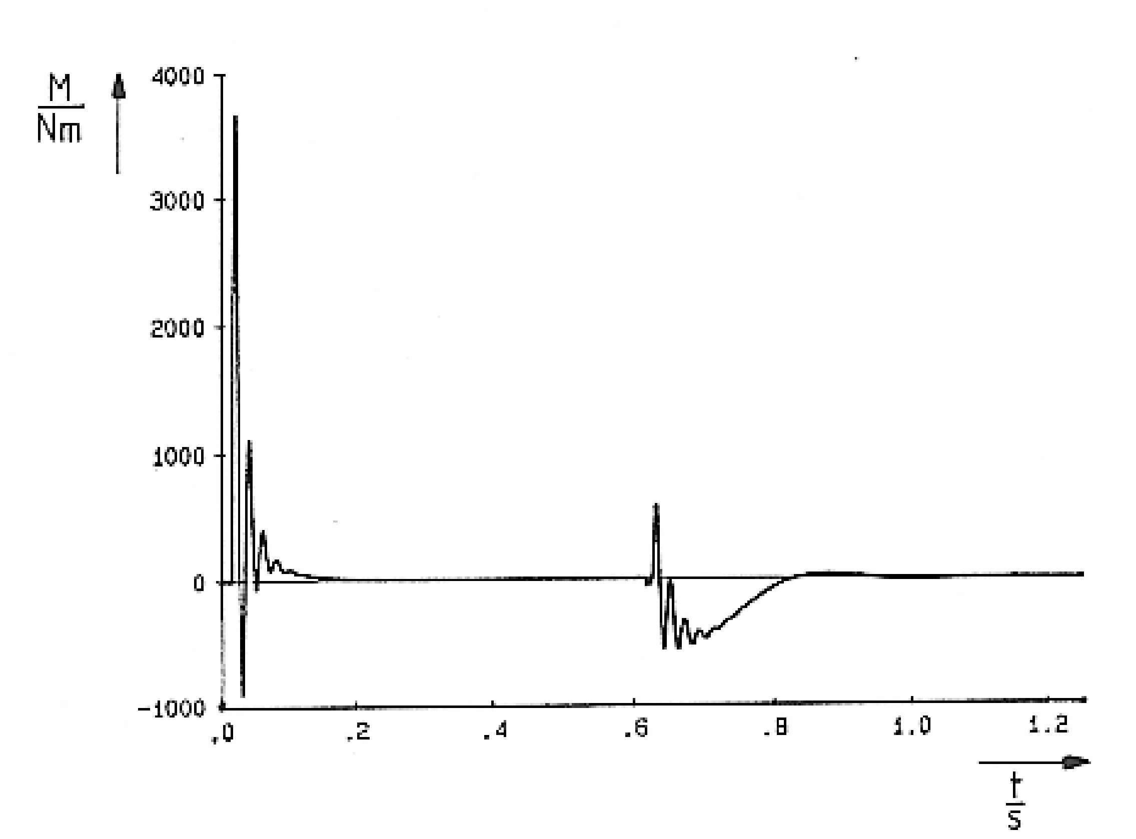


Fig. 81: salient pole machine., sudden short-circuit and coarse synchronization.: torque *M*

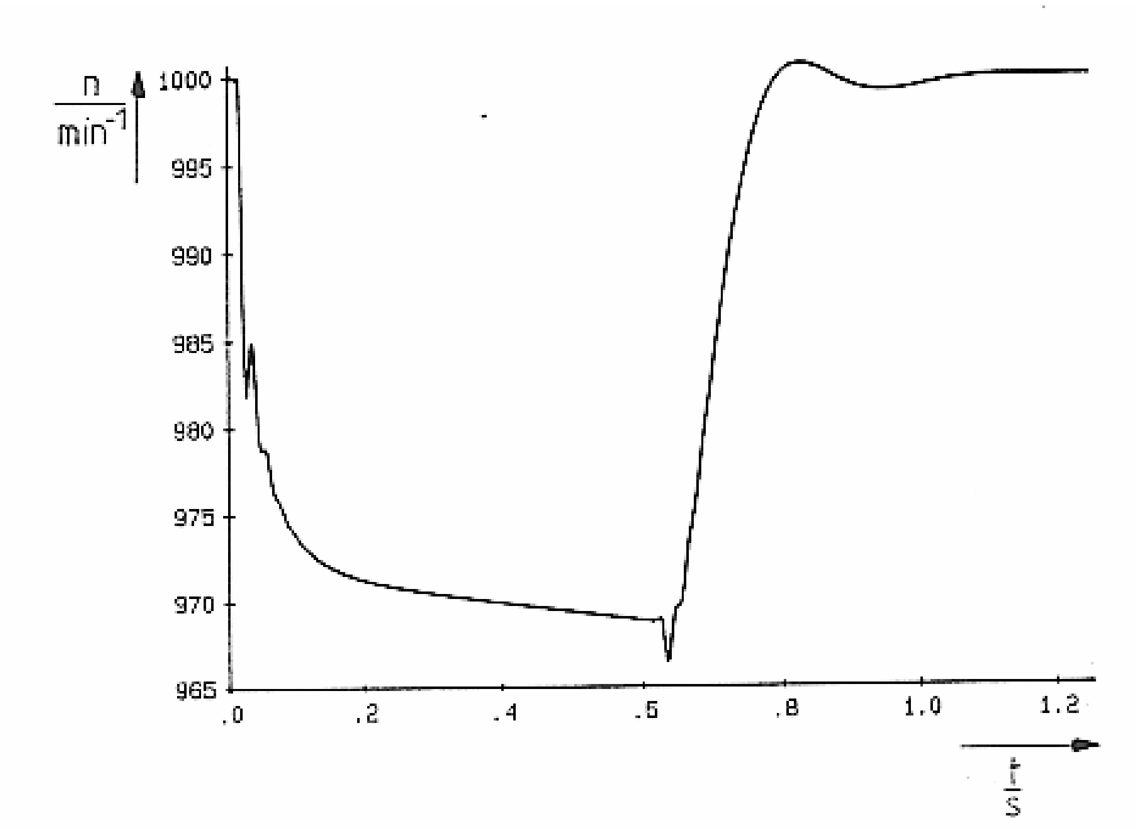


Fig. 82: salient pole machine., sudden short-circuit and coarse synchronization.: speed *n*

Summarized description of the behaviour of salient pole synchronous machines in coarse synchronization and sudden short-circuit operation:

- Because the switching instant was at the zero voltage crossing of phase U, the stator current i_u contains an extended DC component. The time constant for the slope of the sustained short-circuit current is $T_d^{''}$ respectively $T_d^{'}$.
- The excitation current jumps to a value, which is $\frac{1}{s}$ times the no-load value, and falls off corresponding to the stator current. DC components of stator currents cause an alternating component of the excitation current.
- The transformed stator currents i_d and i_q are DC currents in steady-state operation. i_d represents the reactive component, which is responsible for the magnetization, i_q represents the active component, which generates the torque.
- The damper currents i_D and i_Q are only effective at first instant after switching, anytime else they are equal zero.
- Even though the driving torque remains constant, the speed declines after short-circuiting because of the ohmic losses in the resistances.

After the sudden short-circuit the machine was coarse synchronized by reconnecting to the power supply and again transient phenomena occur with high current- and torque-peaks.

4.6 Transient operation of salient-pole machines

Previous sections show, how the system of differential equations of synchronous machines can be solved to calculate the dynamic behavior using analytical or numerical methods.

Using simplifying assumptions, dynamic transient phenomena can be handled in analogy to steady-state phenomena. In this case non-linear system of equations are not necessarily subject of integration:

1.) It is assumed, that the speed of the machine is constant during the transient phenomenon. The bigger the moment of inertia of the rotating machine, the better this assumption is fulfilled.

2.) The effect of the damper winding is not taken into consideration, because the subtransient phenomenon declines rapidly. In lots of cases there is no damper winding at all.

3.) The *induced* voltage components in the voltage equations can be neglected in contrast to the *rotary* induced voltage $(wT >> 1)$. Therefore the DC components of the currents do not occur.

4.) The excitation flux-linkage during the transient phenomenon is assumed to be constant, i.e. the excitation current changes according to the stator current. This assumption is fulfilled, if the resistance of the excitation winding is very small $(R_F = 0)$, because the short-circuited winding keeps its flux constant $\frac{d\Psi_F}{dt} = 0$ and $\Psi_F = const$. The assumption can be fulfilled too, if the machine is equipped with a voltage controller to compensate the ohmic voltage drop if the current changes, i.e. $u_F - R_F \cdot i_F = 0 = \frac{d\Psi_F}{dt}$.

5.) The resistance of the stator winding can be neglected: $R_1 = 0$

6.) There is no zero phase-sequence system.

With these assumptions the system of dynamic equations is simplified as follows:

- voltages:

$$u_d = w \cdot \Psi_q \tag{4.149}$$

$$u_q = -w \cdot \Psi_d \tag{4.150}$$

$$u_F^{'} = 0 \tag{4.151}$$

- currents:

$i_D^{'}$ $\Rightarrow$ does not appear!

$i_Q^{'}$ $\Rightarrow$ does not appear!

- torque

  $$M_W = p \cdot \left(\Psi_q \cdot i_d - \Psi_d \cdot i_q \right) \qquad (4.152)$$

- flux linkages

  $$\Psi_d = L_d \cdot i_d + L_{hd} \cdot i_F^{'} \qquad (4.153)$$

  $$\Psi_F^{'} = L_{hd} \cdot i_d + L_F^{'} \cdot i_F^{'} \qquad (4.154)$$

  $\Psi_D^{'}$ $\Rightarrow$ does not appear!

  $$\Psi_q = L_q \cdot i_q \qquad (4.155)$$

  $\Psi_Q^{'}$ $\Rightarrow$ does not appear!

Flux linkages Ψ_d and Ψ_q are inserted into the voltage equations and the inverse transformation with $a_0 = -\frac{p}{2} + J$ is to be executed according to section 4.2.

$$\underline{U}_d = \frac{u_d}{\sqrt{3}} \cdot e^{ja_0} = \frac{w \cdot L_q \cdot i_q}{\sqrt{3}} \cdot e^{-j\frac{p}{2}} \cdot e^{jJ} = -jX_q \cdot \underline{I}_q \qquad (4.156)$$

$$\underline{U}_q = j\frac{u_q}{\sqrt{3}} \cdot e^{ja_0} = j\frac{-w \cdot \left(L_d \cdot i_d + L_{hd} \cdot i_F^{'} \right)}{\sqrt{3}} \cdot e^{-j\frac{p}{2}} \cdot e^{jJ} = -jX_d \cdot \underline{I}_d - jX_{hd} \cdot \underline{I}_F^{'} \qquad (4.157)$$

whereas

$$\underline{I}_q = \frac{i_q}{\sqrt{3}} \cdot e^{jJ} \qquad (4.158)$$

$$\underline{I}_d = -j\frac{i_d}{\sqrt{3}} \cdot e^{jJ} \qquad (4.159)$$

$$\underline{I}_F^{'} = -j\frac{i_F^{'}}{\sqrt{3}} \cdot e^{jJ} \qquad (4.160)$$

besides

$$\underline{U}_p = -jX_{hd} \cdot \underline{I}_F^{'} \qquad (4.161)$$

The inverse transformation is done for the excitation flux in the direct axis in the same way:

$$\underline{\Psi}_F^{'} = \frac{y_F^{'}}{\sqrt{3}} \cdot e^{ja_0} = \frac{L_{hd} \cdot i_d + L_F^{'} \cdot i_F^{'}}{\sqrt{3}} \cdot (-j) \cdot e^{jJ} = L_{hd} \cdot \underline{I}_d + L_F^{'} \cdot \underline{I}_F^{'} \qquad (4.162)$$

The index “0” describes the state before switching. If the excitation flux-linkage has to be constant after switching, then it has to be:

$$\underline{\Psi}_F^{'} = \underbrace{L_{hd} \cdot \underline{I}_d + L_F^{'} \cdot \underline{I}_F^{'}}_{after\ switching} = \underbrace{L_{hd} \cdot \underline{I}_{d0} + L_F^{'} \cdot \underline{I}_{F0}^{'}}_{before\ switching} = const. \qquad (4.163)$$

This means, that the excitation current changes according to the stator current:

$$\underline{I}_F' = \underline{I}_{F0}' + \frac{\underline{I}_{d0} - \underline{I}_d}{1 + s_2} \tag{4.164}$$

Therefore the voltage equations in the quadrature-axis are:

$$\underline{U}_q = -jX_d \cdot \underline{I}_d - jX_{hd} \cdot \underbrace{\left(\underline{I}_{Fo}' + \frac{\underline{I}_{d0} - \underline{I}_d}{1 + s_2} \right)}_{\underline{I}_F'} \tag{4.165}$$

$$= jX_{hd} \cdot \underline{I}_{F0}' - jX_d \cdot \underline{I}_d \cdot \left(1 - \frac{1}{(1 + s_1)(1 + s_2)} \right) - \frac{jX_d \cdot \underline{I}_{d0}}{(1 + s_1)(1 + s_2)} \tag{4.166}$$

$$= \underline{U}_{p0} - jX_d' \cdot \underline{I}_d - j(1 - s)X_d \cdot \underline{I}_{d0} \tag{4.167}$$

$\underline{U}_{p0}$ is the synchronous generated voltage before switching, $X_d' = s \cdot X_d$ is the transient reactance.

With that the transient synchronous generated voltage $\underline{U}'_p$ can be defined, therefore remaining constant after switching still after switching:

1.) $$\underline{U}_p' = \underline{U}_{p0} - j(1 - s)X_d \cdot \underline{I}_{d0} = \underline{U}_q + X_d' \cdot \underline{I}_d = const \tag{4.168}$$

The synchronous generated voltage changes according to the excitation current:

$$\underline{U}_p = -jX_{hd} \cdot \underline{I}_F' = -jX_{hd} \cdot \left(\underline{I}_{Fo}' + \frac{\underline{I}_{d0} - \underline{I}_d}{1 + s_2} \right) \tag{4.169}$$

$$= -jX_{hd} \cdot \underline{I}_{F0}' - \frac{jX_d \cdot \underline{I}_{d0}}{(1 + s_1)(1 + s_2)} + \frac{jX_d \cdot \underline{I}_d}{(1 + s_1)(1 + s_2)} \tag{4.170}$$

$$= \underline{U}_{p0} - j(1 - s)X_d \cdot \underline{I}_{d0} + j(1 - s)X_d \cdot \underline{I}_d \tag{4.171}$$

2.) $$\underline{U}_p = \underline{U}_p' + j(1 - s)X_d \cdot \underline{I}_d \tag{4.172}$$

The direct-axis voltage is still:

3.) $$\underline{U}_d = -jX_q \cdot \underline{I}_q \tag{4.173}$$

Aspects 1), 2) and 3) describe the transient operation of a salient-pole machine, i.e. the transient phenomenon after switching. The following cases need to be distinguished:

mode of operation	physical	synchr. gen. voltage	direct axis reactance
steady state	$I_F = const$	$U_p = const$	X_d
transient	$\Psi_F = const$	$U_p' = const$	X_d'

Now the U_p is not constant anymore regarding the phasor diagram for the transient operation, but $U_p^{'} = const$. The voltage U_p changes according to I_d. The value of $U_p^{'}$ can be determined using the initial conditions before switching.

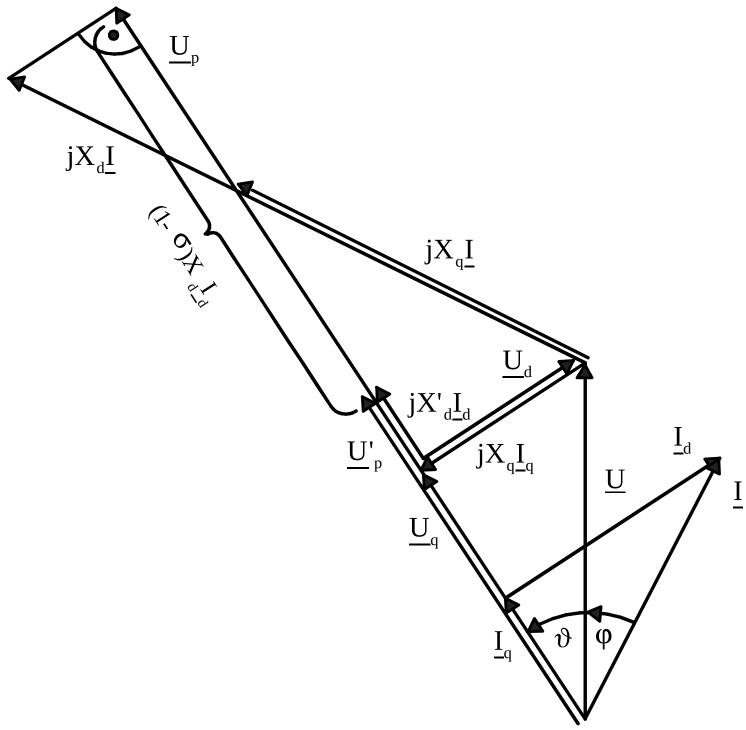


Fig. 83: phasor diagram for the transient operation

Torque in transient operation ensues to:

$$M = \frac{3p}{\omega} \cdot \left(U_d \cdot I_d + U_q \cdot I_q\right) \tag{4.174}$$

with replacements due to:

$$I_d = \frac{U_p^{'} - U_q}{X_d^{'}} \qquad\qquad I_q = \frac{U_d}{X_q} \tag{4.175 a,b}$$

$$U_d = U \cdot \sin\vartheta \qquad\qquad U_q = U \cdot \cos\vartheta \tag{4.176 a,b}$$

Using insertion, the following equations are obtained:

$$M = \frac{3p}{\omega} \cdot \left[\frac{U_p^{'} - U \cdot \cos\vartheta}{X_d^{'}} \cdot U \cdot \sin\vartheta + \frac{U \cdot \sin\vartheta}{X_q} \cdot U \cdot \cos\vartheta \right] \tag{4.177}$$

$$= \frac{3p}{\omega} \cdot \left[\frac{U_p^{'} \cdot U}{X_d^{'}} \cdot \sin\vartheta + \left(\frac{1}{X_q} - \frac{1}{X_d^{'}} \right) \cdot \frac{U^2}{2} \cdot \sin 2\vartheta \right] \tag{4.178}$$

An equation for the torque in transient operation is achieved, which shows nearly the same structure like the equation for the torque in steady-state operation but U_p is replaced by $U_p^{'}$ and X_d is replaced by $X_d^{'}$.

4.6.1 Power supply operation

There are two kinds of load cases, which cause a transient phenomenon, if the machine is connected to the power supply:

1.) The electrical sudden load change, caused by a switching in the power supply network. The supply voltage and the supply reactance are suddenly changed. Because $U_p^{'}$ has to be constant, the currents I_d and I_q change stepwise, because the supply voltage and the supply frequency are suddenly changed. Therefore the generated torque is changed too. Instantaneously after switching, J is equal J_0, because the speed does not change immediately due to the moment of inertia. The speed of the machine does not change fast, therefore there is a surplus/shortfall of torque, which causes a motion process $J(t)$ of the rotor. The resulting currents and the time characteristic of the torque in transient operation can be determined using the precondition $U_p^{'} = const$.

2.) The mechanical sudden load change, caused by a stepwise change of the driving torque. The supply voltage and the supply reactance do not change in this case. Shortly after the sudden load change, the same currents flow as before. Therefore the synchronous machine generates the same torque as before. The surplus/shortfall of torque causes a motion process $J(t)$ of the rotor. Therefore the currents and the torque of the synchronous machine change. Their time characteristic can be determined using the precondition $U_p^{'} = const$.

After the declination of the transient phenomenon, a new steady-state operating state adjusts itself gradually.

For easier description the following base values (so called “per-unit quantities”) are to be used:

$$u = \frac{U}{U_{NStr}} \tag{4.179}$$

$$i = \frac{I}{I_{NStr}} \tag{4.180}$$

$$i_F = \frac{I_F}{I_{F0}} = \frac{U_p}{U_{NStr}} \tag{4.181}$$

$$r = \frac{R}{U_{NStr}/I_N} \tag{4.182}$$

$$x = \frac{X}{U_{NSr}/I_N} \tag{4.183}$$

$$m = \frac{M}{M_N} = \frac{M}{\frac{3p}{w} \cdot U_{NStr} \cdot I_N \cdot \cos j_N} = \frac{M}{M_{SN} \cdot \cos j_N} \tag{4.184}$$

- characteristic of the excitation current:
  - o steady-state $i_F = 1$ (4.185)
  - o transient $i_F = u'_p + (1 - s) \cdot x_d \cdot i_d$ (4.186)

$$i_d = \frac{u'_p - u \cdot \cos J}{x'_d} \tag{4.187}$$

$$i_F = \frac{u'_p}{s} - \frac{1-s}{s} \cdot u \cdot \cos J \tag{4.188}$$

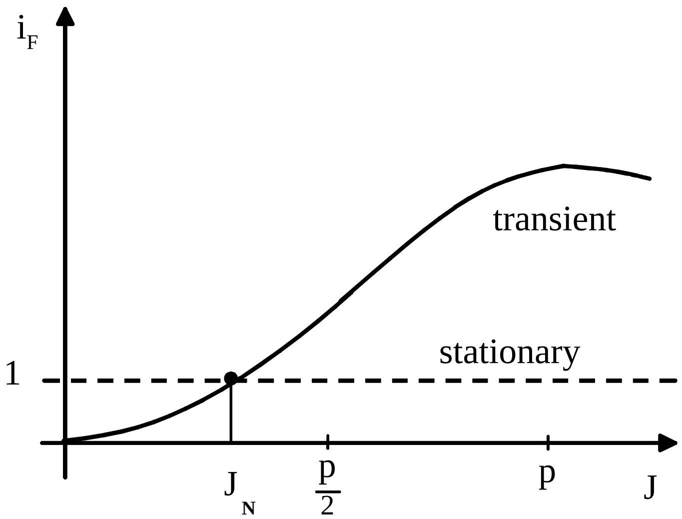


Fig. 84: current i_F versus J

- characteristic of the torque
  - o steady-state $m = \frac{1}{\cos j_N} \cdot \left[\frac{u_p}{x_d} \cdot \sin J + \frac{1}{2} \cdot \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \cdot \sin 2J \right]$ (4.189)
  - o transient $m = \frac{1}{\cos j_N} \cdot \left[\frac{u'_p}{x'_d} \cdot \sin J + \frac{1}{2} \cdot \left(\frac{1}{x_q} - \frac{1}{x'_d} \right) \cdot \sin 2J \right]$ (4.190)

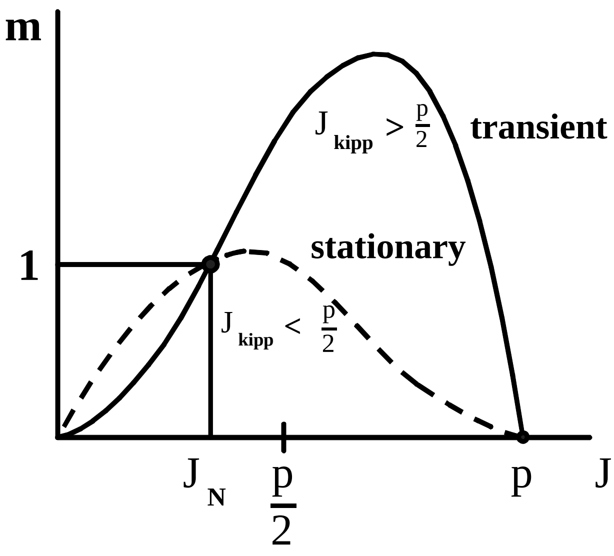


Fig. 85: torque m vs. J

Because of the constant flux in the excitation winding, the machine compounds itself during the transient phenomenon by increasing the excitation current and with it the torque on a multiple of the rated values. Therefore the dynamic stability is increased to angular displacements of more than 90° (electrical).

The transient phenomenon of a sudden load change in power supply operation and the self-compounding because of the constant excitation flux can be illustrated with the phasor diagram.

1. initial state for the steady-state operation: $u = 1,\ i = 1,\ j = j_N$ (4.191)

   draw phasor diagram : $u_p = u_q + x_d \cdot i_d$ (4.192)

   $u_p^{'} = u_q + x_d^{'} \cdot i_d$ (4.193)

2. transient operation: $u_p^{'} = const,\ J > J_N,\ u = 1$ (4.194)

Draw direction of J, take out (=read, measure) u_d and u_q, calculate:

$$i_q = \frac{u_d}{x_q} \quad (4.195)$$

$$i_d = \frac{u_p^{'} - u_q}{x_d^{'}} \quad (4.196)$$

$$\underline{i} = \underline{i}_d + \underline{i}_q \quad (4.197)$$

Thus follows: $u_p = u_p^{'} + (1 - s_{dF}) \cdot x_d \cdot i_d$ (4.198)

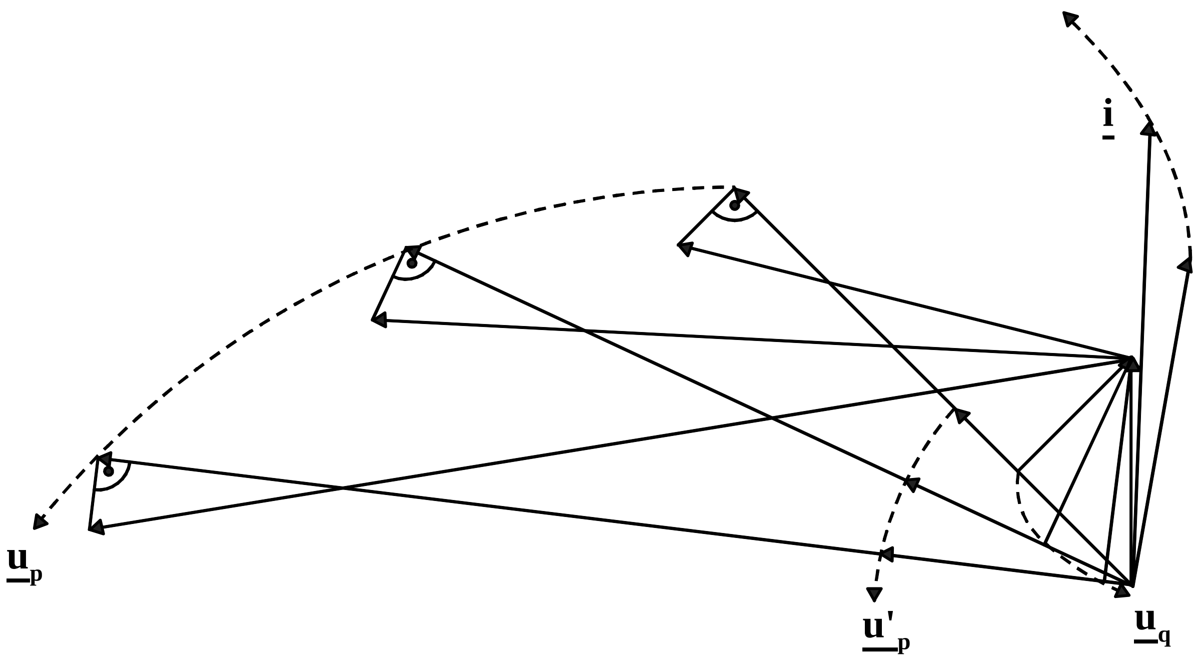


Fig. 86: phasor diagram

4.6.2 Solitary operation

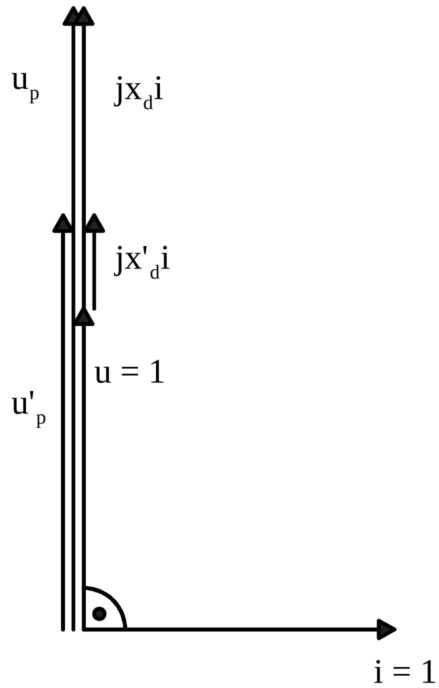
The transient theory can also be applied for isolated operation. Again applying $U_p^{'} = const$

$U_p^{'}$ is determined from the instant before switching. Because of the constant flux in the excitation winding, the currents and voltages instantaneously after switching result from $U_p^{'} = const$.

After a certain period time, if the transient phenomenon has fallen off, $U_p = const$ is applied. For this reason the currents and voltages for the most recent steady-state can be determined.

Between initial and final state there is a transient phenomenon with T_{d0} (T_{F0}) if the stator winding is open-circuited or short circuited, respectively T_{dK} (T_{FK}) if the excitation winding is open-circuited or short circuited.

Example: Synchronous machine connected to power supply; rated current and $\cos j = 0$ (inductive) assumed.



Values:

- $u = 1$, $i = 1$, $j = 90°$
- $u_q = 1$, $u_d = 0$
- $i_q = 0$, $i_d = 1$
- $u_p = 1 + x_d = i_F$
- $u_p^{'} = 1 + x_d^{'}$

are obtained from the phasor diagram (Fig. 87) at steady-state operation before switching.

Fig. 87: synchronous machine, phasor diagram

- after switching applies:

$$i = 0 \text{ and } u_p^{'} = const.$$

- instantaneously after switching we obtain for the transient operation:

$$u = u_p^{'} + x_d^{'} \cdot \underbrace{i_d}_{=0} \tag{4.199}$$

- after a long period of time we obtain for the new steady-state operation:

$$u = u_p + x_d \cdot \underbrace{i_d}_{=0} \tag{4.200}$$

If the stator winding is open-circuited, the time constant ensues to:

$$\textit{time characteristic} = \textit{final state} + (\textit{initial state} - \textit{final state}) \cdot e^{\frac{-t}{\textit{time const.}}} \tag{4.201}$$

So that the transient phenomenon can be described as:

$$u = u_p + \left(u_p^{'} - u_p\right) \cdot e^{-\frac{t}{T_{F0}}} \tag{4.202}$$

The discussed time characteristic is shown in the diagram of Fig. 88:

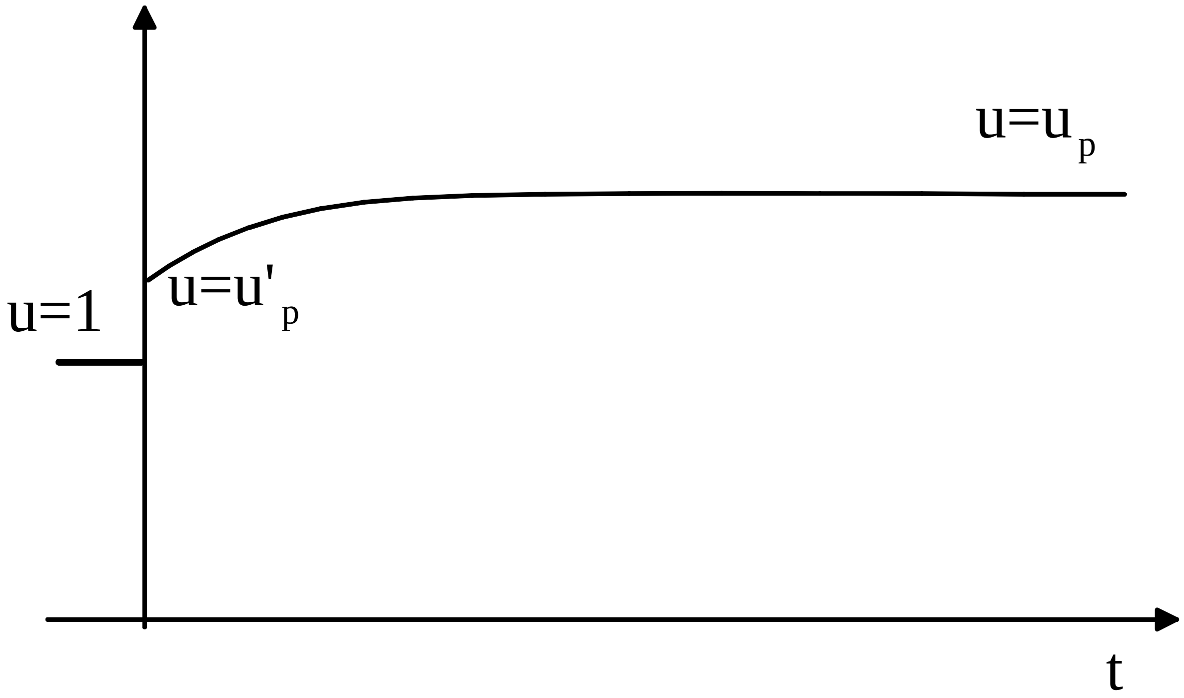


Fig. 88: time characteristic

4.6.3 Summary and conclusion of transient operation

Three equations are basically used:

$$U_p^{'} = U_q + X_d^{'} \cdot I_d \tag{4.203}$$

$$U_p = U_p^{'} + \left(1 - s_{dF}\right) \cdot X_d \cdot I_d = U_q + X_d \cdot I_d \tag{4.204}$$

$$U_d = X_q \cdot I_q \tag{4.205}$$

4.6.4 Scheme for transient operation

1.) Steady-state operation before switching,
phasor diagram, determination of $U_p^{'}$

2.) Transient operation instantaneously after switching,
calculate new currents, voltages and torque using $U_p^{'} = const.$

3.) Steady-state operation after a long period of time
phasor diagram, determination of U_p, calculate currents, voltages and torque after that

4.) Transient phenomenon in between: e-function
time constant:

$$\left.\begin{matrix} T_{d0} \\ T_{dK} \end{matrix}\right\}\text{stator}\left\{\begin{matrix} \text{field open-circuited} \\ \text{field short-circuited} \end{matrix}\right.$$

<u>case:</u> switching appears in field.

$$\left.\begin{matrix} T_{F0} \\ T_{FK} \end{matrix}\right\}\text{Feld}\left\{\begin{matrix} \text{stator open-circuited} \\ \text{stator short-circuited} \end{matrix}\right.$$

<u>case:</u> switching appears in stator

5 Servo-motor

Permanent-field synchronous machines with rotor position encoder are also known as servo-motors.

5.1 General design and function

Design:

The stator of this machine type features a conventional three-phase winding. The permanent-field rotor is equipped with rare-earth or ferrite magnets. The power inverter is controlled by the rotor position encoder.

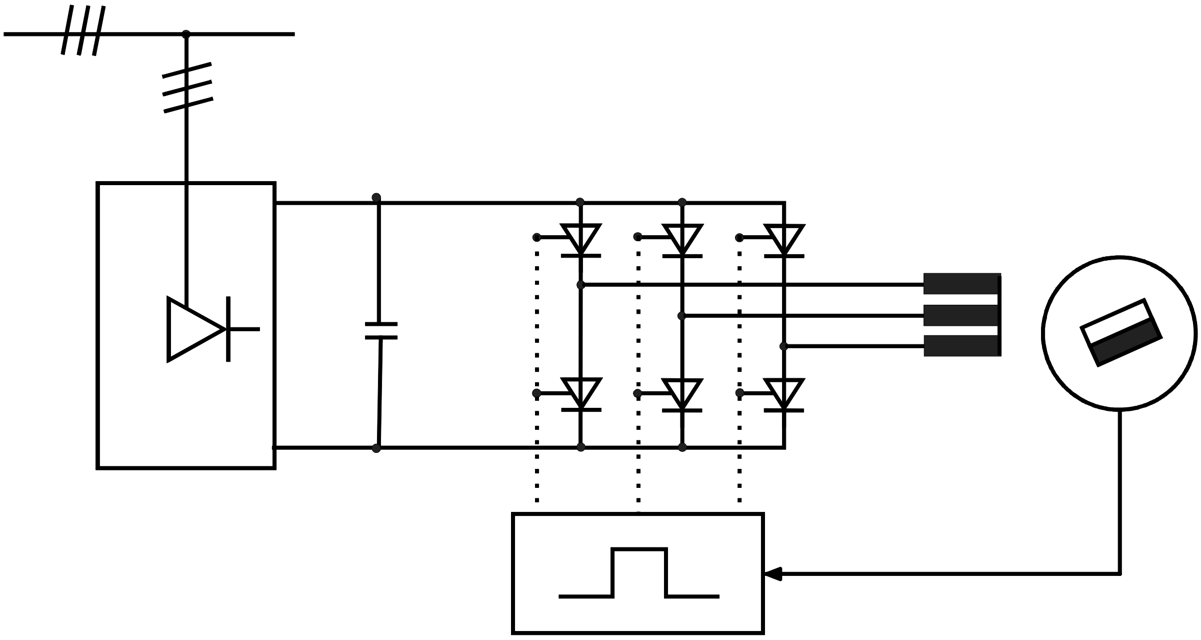


Fig. 89: permanent-field synchronous machine (servo motor)

Function:

The three-phase winding of the stator is supplied with a square-wave or sinusoidal three-phase system depending on the rotor position. Thus a rotating m.m.f is caused, which rotates with the exact rotor speed and which generates a time constant torque in co-action with the magnetic field of the permanent-field rotor. The rotating field in the stator is commutated depending on the rotor position in such a manner, that the rotating m.m.f in the stator and the rotor field are perpendicular with a constant electrical angle of 90°.

Thus results an operating method, which does not correspond to the operating method of synchronous machines anymore, but to that of DC machines. Here the armature current linkage and the excitation field are also perpendicular with a constant electrical angle of 90°. This constant angle is adjusted mechanically with the commutator of the DC machine.

The constant angle of a permanent-field synchronous machine with rotor position encoder is adjusted electrically using a power inverter. This machine type can not fall out of synchronism anymore and behaves like a DC machine. That is why the machine is also called "electronically commutated DC motor (EC-motor)".

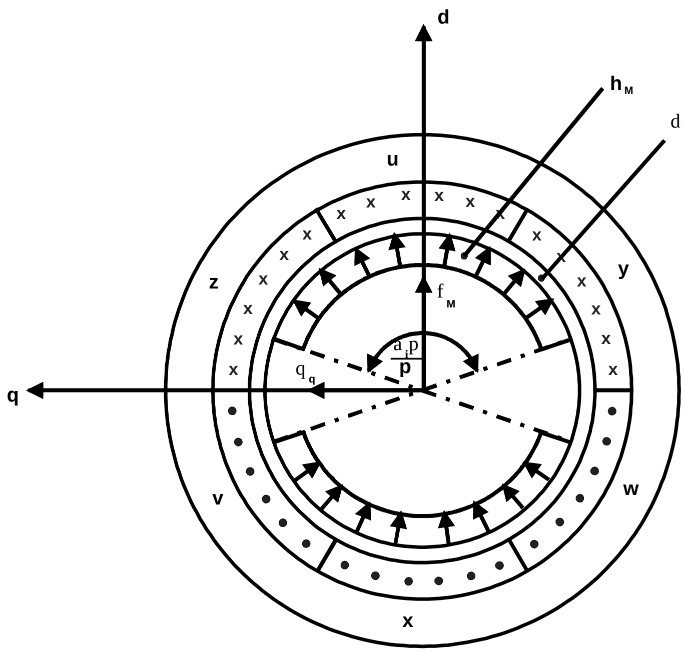


Fig. 90: servo-motor, direction convention

EC-motors are used in machine tools and robotic devices because of their excellent dynamic behavior simple controllability. The brushless technique has a low rate of wear and is maintenance-free.

5.2 Dynamic set of equations

Focus is put on salient-pole machines with magnetic asymmetry $L_q \neq L_d$ in the following. The arrangement of permanent magnets can be reduced to $a_i < 1$ if the machine is supplied with sinusoidal currents, to save permanent magnet material and to improve the voltage waveform. The permanent magnets are magnetizing in the direct axis. The magnetic air-gap in the q-axis is assumed to be equal to the mechanical air-gap. The magnetic air-gap in the d-axis is $d + h_M$ and therefore it is $L_q > L_d$. This effect can be utilized to reduce the current, as to be seen later on. Figure 90 shows a two-pole model.

In case of no damper windings, permanent-field synchronous machines only have three windings to be considered. This applies, if the rotor is made of laminated sheets and if the conductive rare-earth magnets (SmCo, NdFeB) are subdivided into small laminations, to avoid eddy currents.

The following denominations are used:

- d: stator direct axis
- q: stator quadrature axis
- F’: rotor direct axis, permanent magnets

Due to $L_q \neq L_d$ a suitable coordinate system is to be chosen, requiring to rotate with the rotor,

$$\frac{da}{dt} = \frac{dg}{dt}. \tag{5.206}$$

Practically the previously discussed coordinate system for salient-pole machines without damper windings is used. The load reference arrow system for motor operation is applied on the considered case.

There are only voltage equations and the flux-linkage of the stator winding, but no voltage equation for the permanent magnets. The constant rotor flux Ψ_M in the d-axis, which is caused by the permanent magnets, is considered by a constant equivalent excitation current $i_{F0}^{'}$ in the equation for the stator flux-linkage in the d-axis. The stator winding has no effect on the permanent magnets.

$$u_d = R_1 \cdot i_d + \frac{d\Psi_d}{dt} - w \cdot \Psi_q \tag{5.1}$$

$$u_q = R_1 \cdot i_q + \frac{d\Psi_q}{dt} + w \cdot \Psi_d \tag{5.2}$$

$$M_{el} = p \cdot \left(\Psi_d \cdot i_q - \Psi_q \cdot i_d\right) = \frac{J}{p} \cdot \frac{dw}{dt} + M_W \tag{5.3}$$

$$\Psi_d = L_d \cdot i_d + \underbrace{L_{hd} \cdot i_{F0}^{'}}_{\Psi_M} \tag{5.4}$$

$$\Psi_q = L_q \cdot i_q \tag{5.5}$$

The flux-linkages can be directly pasted into the stator voltage equation and into the torque equation.

$$u_d = R_1 \cdot i_d + L_d \cdot \frac{di_d}{dt} - w \cdot L_q \cdot i_q \tag{5.6}$$

$$u_q = R_1 \cdot i_q + L_q \cdot \frac{di_q}{dt} + w \cdot L_d \cdot i_d + w \cdot L_{hd} \cdot i_{F0}^{'} \tag{5.7}$$

$$M_{el} = p \cdot \left(L_{hd} \cdot i_{F0}^{'} \cdot i_q + L_d \cdot i_d \cdot i_q - L_q \cdot i_q \cdot i_d\right) \tag{5.8}$$

$$= p \cdot \left[L_{hd} \cdot i_{F0}^{'} - i_d \cdot \left(L_q - L_d\right)\right] \cdot i_q = \frac{J}{p} \cdot \frac{dw}{dt} + M_W \tag{5.9}$$

5.3 Steady state operation

In steady-state operation the flux-linkages in the rotating system and the speed are constant, i.e.

- $\frac{d\Psi}{dt} = 0$ (5.10)
- $\frac{dw}{dt} = 0$ (5.11)

For this reason the dynamic system of equation can be simplified and the torque equation is decoupled.

$$u_d = R_1 \cdot i_d - w \cdot L_q \cdot i_q \tag{5.12}$$

$$u_q = R_1 \cdot i_q + w \cdot L_d \cdot i_d + w \cdot L_{hd} \cdot i_{F0}^{'} \tag{5.13}$$

$$M_{el} = p \cdot \left[L_{hd} \cdot i_{F0}^{'} - i_d \cdot \left(L_q - L_d\right)\right] \cdot i_q \tag{5.14}$$

For the inverse transformation of the rotating system with

$$\frac{da}{dt} = \frac{dg}{dt} \tag{5.15}$$

the (still) arbitrary integration constant is practically chosen:

$$a_0 = -\frac{p}{2} \tag{5.16}$$

After the inverse transformation into the complex notation, the rotor axis, which is the magnetizing axis of the permanent magnets, is pointing in direction of the negative imaginary axis and the q-axis, where the torque is generated, is orientated to the real axis ($\Rightarrow$ compare with induction machine).

$$\underline{U} = \frac{u_d}{\sqrt{3}} \cdot e^{j \cdot a_0} + j \cdot \frac{u_q}{\sqrt{3}} \cdot e^{j \cdot a_0} = \underline{U}_d + \underline{U}_q \tag{5.17}$$

$$\underline{U}_d = \frac{R_1 \cdot i_d - w \cdot L_q \cdot i_q}{\sqrt{3}} \cdot (-j) = R_1 \cdot \underline{I}_d + j \cdot X_q \cdot \underline{I}_q \tag{5.18}$$

$$\underline{U}_q = j \cdot \frac{R_1 \cdot i_q + w \cdot L_d \cdot i_d + w \cdot L_{hd} \cdot i_{F0}^{'}}{\sqrt{3}} \cdot (-j) = R_1 \cdot \underline{I}_q + j \cdot X_d \cdot \underline{I}_d + j \cdot X_{hd} \cdot \underline{I}_{F0}^{'} \tag{5.19}$$

whereas currents are defined as:

$$\underline{I}_q = \frac{i_q}{\sqrt{3}}, \qquad \underline{I}_d = -j \cdot \frac{i_d}{\sqrt{3}}, \qquad \underline{I}_{F0}^{'} = -j \cdot \frac{i_{F0}^{'}}{\sqrt{3}} \tag{5.20 a-c}$$

With:

$$\underline{U}_p = j \cdot X_{hd} \cdot \underline{I}_{F0}^{'} \tag{5.21}$$

The complex voltage equation is:

$$\underline{U} = R_1 \cdot \left(\underline{I}_d + \underline{I}_q\right) + j \cdot X_q \cdot \underline{I}_q + j \cdot X_d \cdot \underline{I}_d + \underline{U}_p \tag{5.22}$$

Torque is obtained from the root-mean-square values:

$$M_{el} = \frac{3 \cdot p}{w} \cdot \left[w \cdot L_{hd} \cdot \frac{i_{F0}^{'}}{\sqrt{3}} - \frac{i_d}{\sqrt{3}} \cdot \left(w \cdot L_q - w \cdot L_d \right) \right] \cdot \frac{i_q}{\sqrt{3}} \tag{5.23}$$

$$M_{el} = \frac{3 \cdot p}{w} \cdot \left[U_p - I_d \cdot \left(X_q - X_d \right) \right] \cdot I_q \tag{5.24}$$

Based on the knowledge of equations 5.17 - 5.22 the phasor diagram can be drawn. U_p and I_q are in phase. Two modes need to be distinguished:

normal operation $I_d = 0$

$y = J + j = 0°$

field-weakening $I_d < 0$

$y = J + j < 0°$

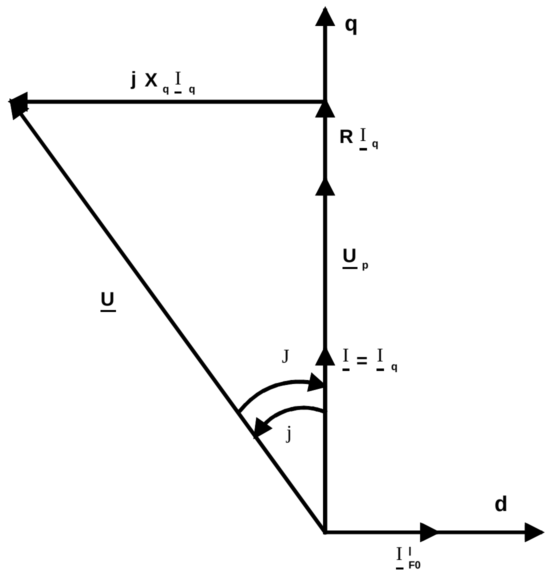


Fig. 91: normal operation

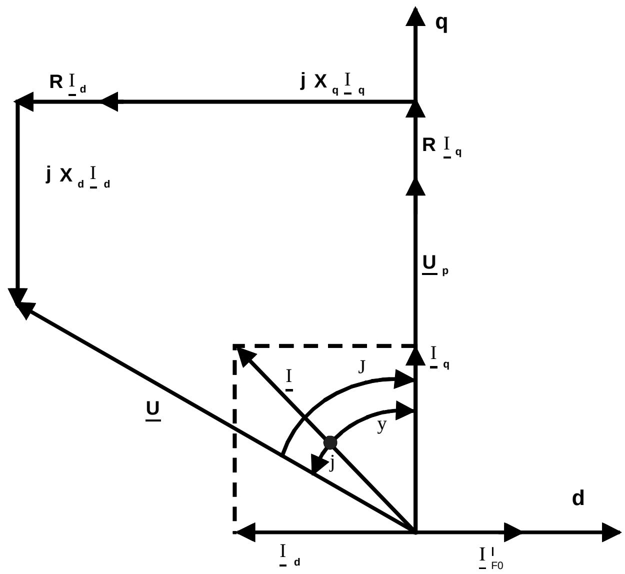


Fig. 92: field weakening

Compared to DC machines, field weakening of permanent-field synchronous machines can only be performed within certain limits, if a negative direct current component is injected additionally to the torque generating current in the q-axis, $y < 0$. Thus the angle of stator current linkage and rotor field is increased to an electrical angle of more than 90°. The speed increases too. The injection of a negative direct current can also be used to minimize the current and to improve the power factor. The precondition is a rotor with a magnetic asymmetry $X_q > X_d$. Then the negative direct current component and the reluctance in conjunction with the quadrature-axis component of the current generate an additional torque contribution.

Thus the total current consumption is decreased. At the same time the phase displacement between terminal voltage and stator current is reduced, so a power factor $\cos j$ of nearly 1 is obtained. This method allows an operation with lower current consumption and therefore with lower copper losses. Thus the utilization of the machine is increased, power of the converter output power is reduced. With increasing difference $\left(X_q - X_d\right)$ this so called pre-control $y < 0$ becomes more and more effective.

The characteristics of the permanent-field synchronous machine with rotor position encoder in steady-state operation can be determined using the direct and quadrature components of the voltage equations and the torque equation.

$$U_d = R_1 \cdot I_d - X_q \cdot I_q \tag{5.25}$$

$$U_q = R_1 \cdot I_q + X_d \cdot I_d + U_p \tag{5.26}$$

$$M = \frac{3 \cdot p}{w} \cdot \left(U_p - I_d \cdot \left(X_q - X_d\right)\right) \cdot I_q \tag{5.27}$$

The frequency-dependency of the reactances and of the synchronous generated voltage is considered by basing on rated frequency.

$$X_d = \frac{w}{w_0} \cdot X_{d0} \tag{5.28}$$

$$X_q = \frac{w}{w_0} \cdot X_{q0} \tag{5.29}$$

$$U_p = \frac{w}{w_0} \cdot U_{p0} \tag{5.30}$$

$$U_d = R_1 \cdot I_d - \frac{w}{w_0} \cdot X_{q0} \cdot I_q \tag{5.31}$$

$$U_q = R_1 \cdot I_q + \frac{w}{w_0} \cdot X_{d0} \cdot I_d + \frac{w}{w_0} \cdot U_{p0} \tag{5.32}$$

$$M = \frac{3 \cdot p}{w} \cdot \left[\left(\frac{w}{w_0} \cdot U_{p0} - I_d \cdot \frac{w}{w_0} \cdot \left(X_{q0} - X_{d0}\right)\right)\right] \cdot I_q \tag{5.33}$$

Finally results:

- speed (shunt characteristic)

$$\frac{n}{n_0} = \frac{w}{w_0} = \frac{U_q - R_1 \cdot I_q}{U_{p0} + X_{d0} \cdot I_d} \tag{5.34}$$

- torque (shunt characteristic)

$$M = \frac{3 \cdot p}{w_0} \cdot \left[U_{p0} - I_d \cdot \left(X_{q0} - X_{d0}\right)\right] \cdot I_q \tag{5.35}$$

- control instruction (electrical commutator):

$$U_d = R_1 \cdot I_d - \frac{w}{w_0} \cdot X_{q0} \cdot I_q \tag{5.36}$$

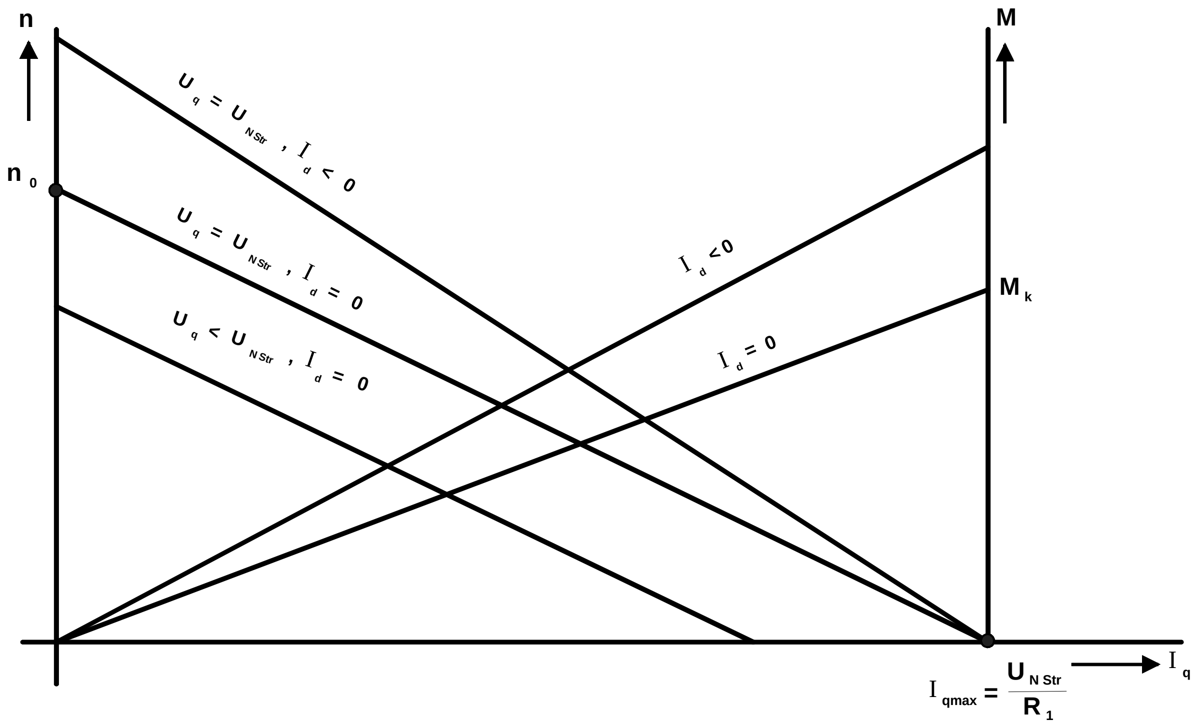


Fig. 93: control instruction, schematic overview

The operational performance of permanent-field synchronous machines with rotor position encoder corresponds to the behavior of separately excited DC machines.

- $U_q \mathrel{\hat{=}} U_A$
- $I_q \mathrel{\hat{=}} I_A$
- $I_d \mathrel{\hat{=}} I_F$
- $U_{p0} = U_{NStr} \mathrel{\hat{=}} k \cdot f \cdot n_0$

By set-point selection of the voltage $U_q \leq U_{NStr}$ the speed can be adjusted non-dissipative in the speed range $n \leq n_0$. By injecting a negative direct current $-I_d$ with a voltage component U_d the speed can be further increased in spite of the permanent field. Torque increases simultaneously.

5.4 Dynamic behavior

With time constants

- $T_d = \frac{L_d}{R_1}$ (5.37)
- $T_q = \frac{L_q}{R_1}$ (5.38)

a system of differential equations is obtained, to be able to completely describe permanent-field synchronous machines.

$$T_d \cdot \frac{di_d}{dt} + i_d = \frac{u_d}{R_1} + w \cdot T_q \cdot i_q \quad (5.39)$$

$$T_q \cdot \frac{di_q}{dt} + i_q = \frac{u_q}{R_1} - w \cdot T_d \cdot i_d - \frac{w \cdot L_{hd}}{R_1} \cdot i'_{F0} \quad (5.40)$$

$$\frac{J}{p} \cdot \frac{dw}{dt} = p \cdot \left[i'_{F0} \cdot L_{hd} - i_d \cdot \left(L_q - L_d \right) \right] \cdot i_q - M_W \quad (5.41)$$

The transformation of the stator voltages to a rotating system with $\frac{da}{dt} = w$ is done as usual:

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = \left[T_a \right] \cdot \left[T_{32} \right] \cdot \begin{bmatrix} u_u \\ u_v \end{bmatrix} \quad (5.42)$$

Based on this, the structure diagram of a complete servo drive can be drawn. Besides the machine model a PWM-converter and the speed control are included.

Thus the machine can be simulated and computational calculated with (spice-oriented-) simulation tools.

excitation values:	u_d,	u_q,	m_W	
state values:	i_d,	i_q,	w	
initial conditions:	$i_d(0) = i_q(0) = w(0) = 0$,			$a_0 = -\frac{p}{2}$

Figure 94 shows the according structure diagram of a permanent-field synchronous machine with rotor position encoder (top left: converter; top right: control; bottom: machine).

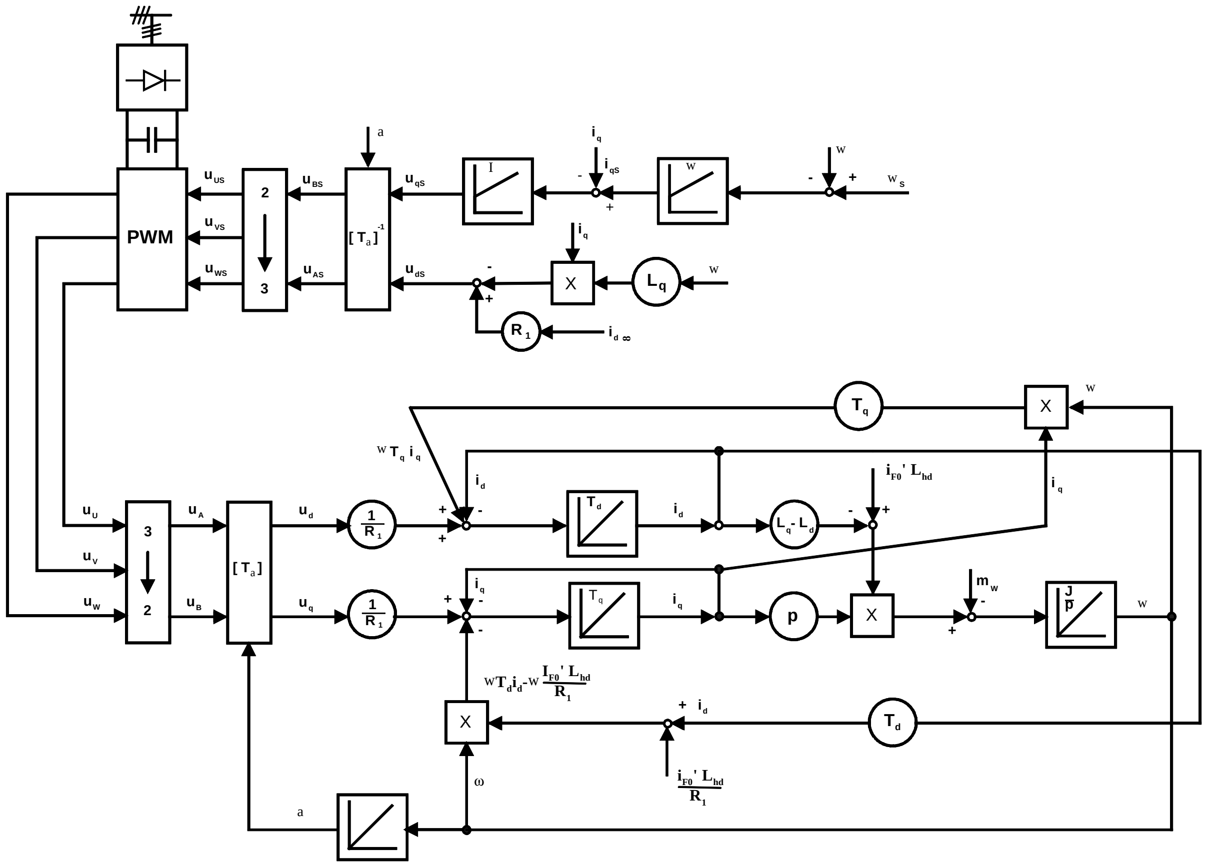


Fig. 94: servo-motor drive, structure diagram

The machine is controlled in such a manner, that armature current-linkage and excitation field are perpendicular in normal operation. This means that, if the flux of the permanent magnets is represented by $L_{hd} \cdot i'_{F0}$, the stator current has no direct component $i_d = 0$ and the quadrature component i_q is the torque-generating component.

In order to keep $i_d = 0$, follows for u_d:

$$\frac{u_d}{R_1} + w \cdot T_q \cdot i_q = 0 \tag{5.43}$$

$$u_d = -w \cdot L_q \cdot i_q \tag{5.44}$$

If a control method with $i_d \neq 0$ is applied, for example field-weakening $i_d < 0$, the voltage control in the d-axis has to be:

$$\frac{u_d}{R_1} + w \cdot T_q \cdot i_q = i_{d\infty} \tag{5.45}$$

$$u_d = -w \cdot L_q \cdot i_q + R_1 \cdot i_{d\infty} \tag{5.46}$$

Thus the time characteristic of the direct current ensues to:

$$i_d = i_{d\infty} \cdot \left(1 - e^{-\frac{t}{T_d}}\right) \tag{5.47}$$

Voltage in the quadrature axis results from the speed which is supposed to be adjusted. For example at no-load operation and rated frequency:

$$\frac{u_q}{R_1} - w \cdot \frac{L_{hd}}{R_1} \cdot i'_{F0} = 0 \tag{5.48}$$

$$u_q = w \cdot L_{hd} \cdot i'_{F0} = \sqrt{3} \cdot U_{NStr} \tag{5.49}$$

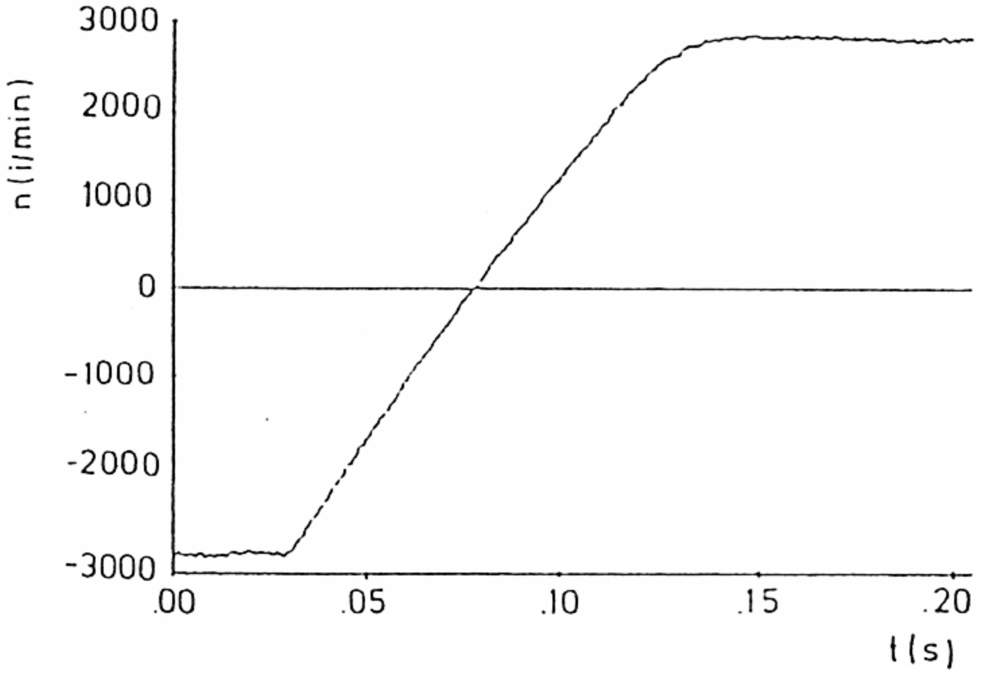


Fig. 95: measured mechanical speed

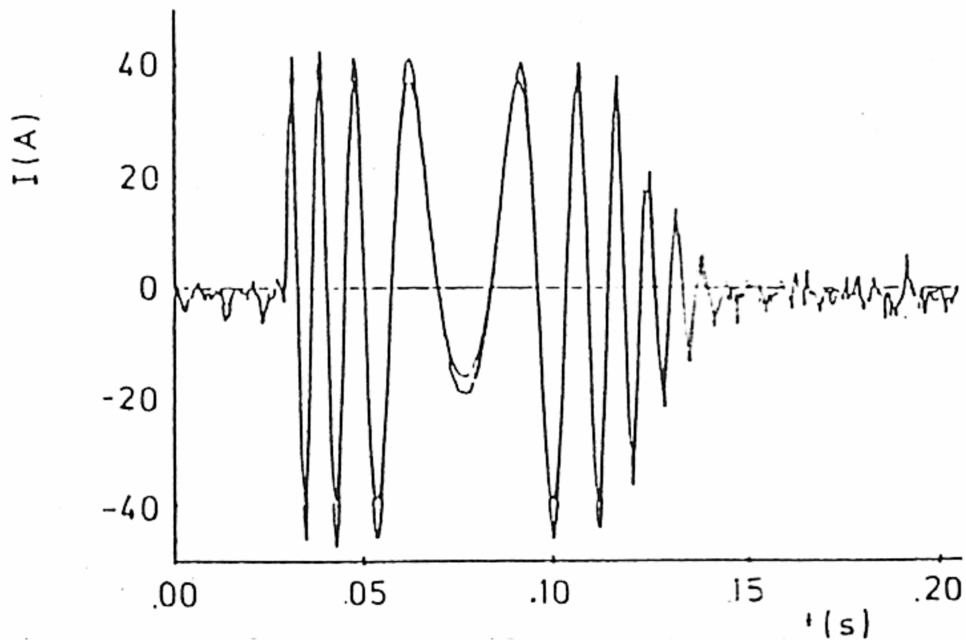


Fig. 96: measured phase-current (setpoint and actual value)

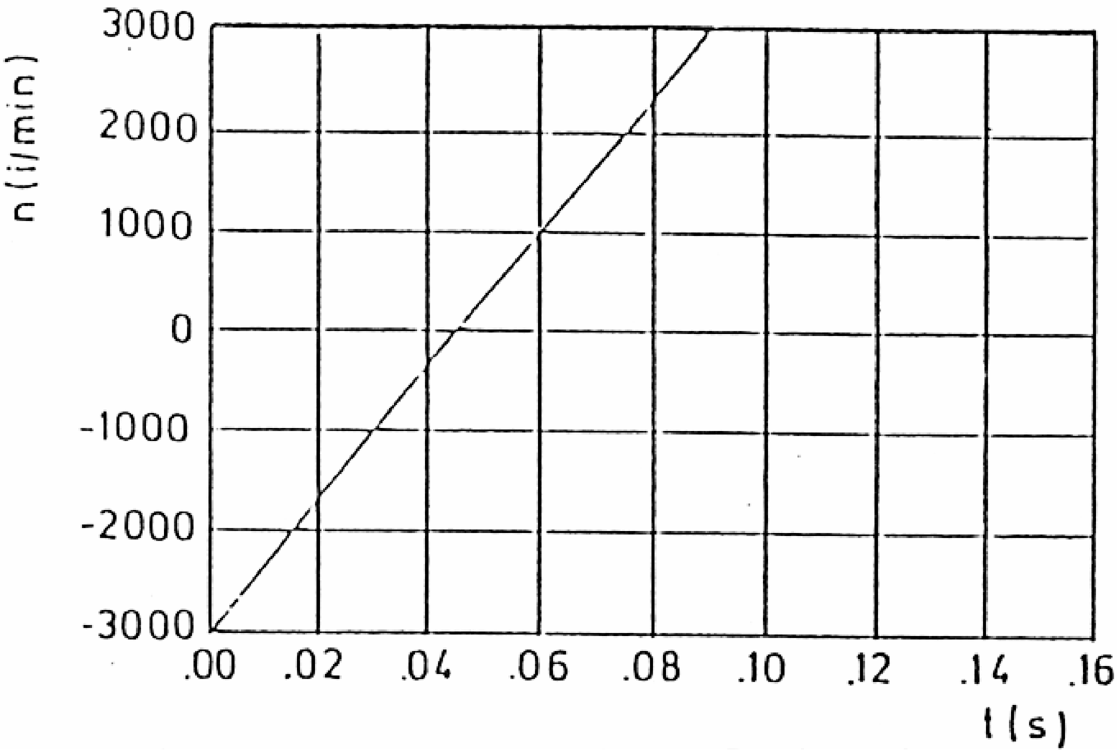


Fig. 97: calculated mechanical speed

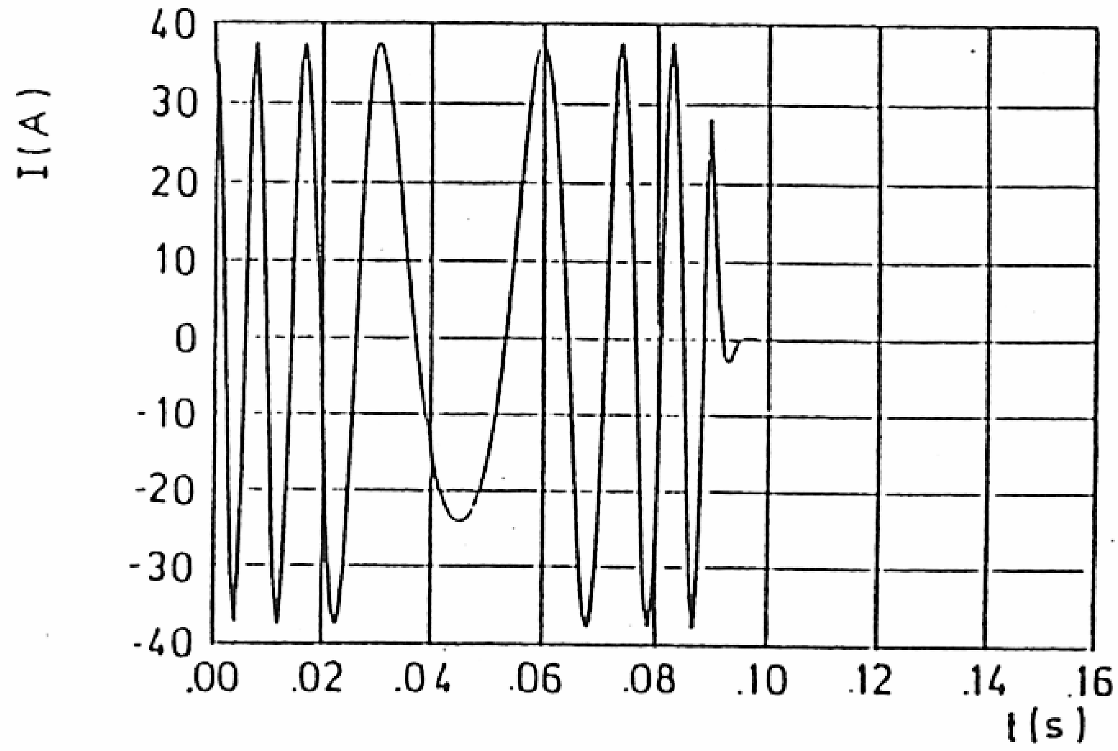


Fig. 98: calculated phase current

Figures show reversing process of a servomotor: comparison of measurement and calculation.

5.5 Voltage- and current waveforms of servo-motors with rotor position encoder

Permanent-field (PM = permanent-magnet, permanent-field) synchronous machines can be operated in different ways. In block-operation, the motor is supplied with rectangular (block) currents and the distribution of the air-gap flux density is rectangular. If the motor is supplied with sinusoidal currents and the rectangular distribution of the air-gap flux density is retained, then we have mixed operation. In sinusoidal operation, the current and the distribution of the air-gap flux are sinusoidal. The figure shows the characteristics of flux density, current and voltage.

If a machine is operated in block-operation, then it is also called brushless or electrically commutated DC machine. If a machine is operated in sinusoidal operation, it is also called self-controlled synchronous machine. The operational performance of permanent-field synchronous machines with rotor position encoder generally corresponds to the operational performance of DC machines.

If the machine is supplied with sinusoidal currents, sinusoidal induced voltages are necessary. We can obtain a nearly sinusoidal air-gap field using parallel magnetized instead of radial magnetized permanent magnets and by designing a suitable stator winding (chording for example).
Another possibility is to supply the machine with rectangular (block) currents. The total supply current has a constant magnitude and is distributed cyclic to the three stator phases, which results in current blocks with an electrical length of 120° and dead times of 60°. If the induced voltage during the length of a current block is constant, then power of the phase is constant too. During the dead times the induced voltage has no influence on the torque generation. The trapezoidal characteristic of the induced voltage results from $q > 1$ and because of the skewing of the stator slots of one slot pitch.

The advantages of the rectangular supply in comparison to the sinusoidal supply are a 15% higher utilization of the machine and the usage of simple position sensors (three photoelectric barriers) instead of expensive resolvers and an easier signal processing.

The disadvantages of the rectangular supply in comparison to the sinusoidal supply are:

- with increasing speed eddy-current losses arise in the conductive rare-earth permanent magnets (in comparison with non-conductive ferrite magnets) caused by the slot harmonics and the jumping rotating m.m.f..
- because of the machine- inductances and the voltage limitation of the converter, there are heavy deviances from the rectangular current form at high speed. The results are a reduced torque and higher losses.
- because of the non-ideal commutation of the phase currents at rectangular supply, angle-dependent huntings occur at lower speed, which has to be compensated by the control.

In contrast the mixed operation has advantages. If the machine is supplied sinusoidal, if it has a rectangular flux-density distribution in the air-gap and if the stator winding is chorded, to achieve a sinusoidal induced voltage, then the best motor utilization is obtained. In this case the fundamental wave of the flux-density in the air-gap is increased and at the same time the losses are reduced. A 26% higher machine utilization can be achieved, compared to sinusoidal supply, respectively 10% higher compared to rectangular supply. The operation with sinusoidal currents requires an exact information of the rotor position, which requires an expensive encoder-system.

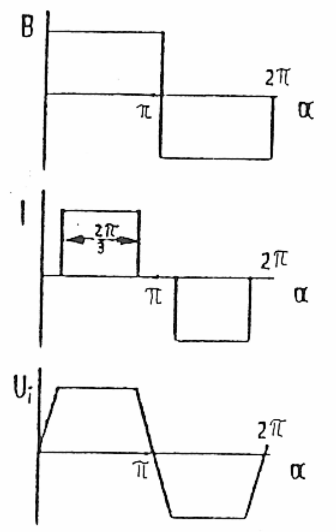
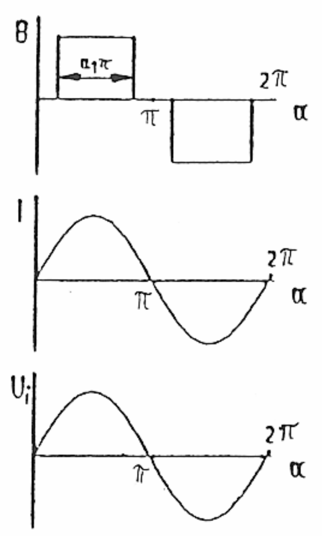
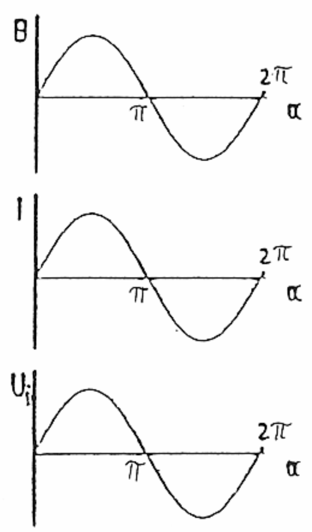
Block (B1), Block (I1) Trapezoidal (Ui)	Block (B1), Sinusoidal (I1), Sinusoidal (Ui)	Sinusoidal (B1), Sinusoidal (I1), Sinusoidal (Ui)
		
$U_{i\sqcap} = 2l(w\xi)BV_A$ $= \omega(w\xi)\frac{2}{\pi}\tau_p l B_{max}$ $= cB_{max}$	$U_{i\sim\sqcap}$ $= \omega(w\xi)\frac{\frac{2}{\pi}\tau_p l B_{max}\frac{4}{\pi}\sin(a_i\frac{\pi}{2})}{\sqrt{2}}$ $= \frac{cB_{max}}{\sqrt{2}}\frac{4}{\pi}\sin(a_i\frac{\pi}{2})$	$U_{i\sim}$ $= \omega(w\xi)\frac{\frac{2}{\pi}\tau_p l B_{max}}{\sqrt{2}}$ $= \frac{cB_{max}}{\sqrt{2}}$
$P_V = 2RI_{\sqcap}^2$ $I = \sqrt{\frac{3}{2}}I_\sim$	$P_V = 3RI_\sim^2$	$P_V = 3RI_\sim^2$
$P_\sqcap = 2U_{i\sqcap}I_\sqcap$	$P_{\sim\sqcap} = 3U_{i\sim\sqcap}I_\sim$	$P_\sim = 3U_{i\sim}I_\sim$
discrete position measurement (3 photoelectric barriers)	continuous position measurement (resolver or incremental encoder)	
$\frac{P_\sqcap}{P_\sim} = \frac{2}{\sqrt{3}} = 1,15$		
$\frac{P_\sqcap}{P_{\sim\sqcap}} = \frac{2}{\sqrt{3}}\frac{1}{\frac{4}{\pi}\sin(a_i\frac{\pi}{2})} = 0,91$ für $a_i = 1$		
$\frac{P_{\sim\sqcap}}{P_\sim} = \frac{P_{\sim\sqcap}}{P_\sqcap}\frac{P_\sqcap}{P_\sim} = \frac{1}{0,91}1,15 = 1,26$ for $a_i = 1$		

Fig. 99: table, comparing waveforms and resulting power values

6 Appendix

6.1 Formular symbols

A	current coverage; area (in general)
a	number of parallel conductors
B	flux density (colloq. induction)
b	width
C	capacity
c	general constant, specific heat
D	diameter, dielectric flux density
d	diameter; thickness
E	electric field strength
e	Euler’s number
F	force; form factor
f	frequency
G	electric conductance, weight
g	fundamental factor, acceleration of gravity
H	magnetic field strength
h	height; depth
I	current; I_w active current; I_B reactive current
i	instantaneous current value
J	mass moment of inertia
j	unit of imaginary numbers
K	cooling medium flow, general constant
k	number of commutator bars; general constant
L	self-inductance; mutual inductance
l	length
M	mutual inductance; torque
m	number of phases, mass
N	general number of slots
n	rotational speed

O	surface, cooling surface
P	active power
p	number of pole pairs; pressure
Q	reactive power; cross section; electric charge
q	number of slots per pole and phase; cross section
R	efficiency
r	radius
S	apparent power
s	slip; coil width; distance
T	time constant; length of period; absolute temperature; starting time
t	moment (temporal); general time variable
U	voltage (steady value); circumference
u	voltage (instantaneous value); coil sides per slot and layer
V	losses (general); volume; magnetic potential
v	speed; specific losses
W	energy
w	number of windings; flow velocity
X	reactance
x	variable
Y	peak value (crest value)
y	variable; winding step
Z	impedance
z	general number of conductors
α	pole pitch factor; heat transfer coefficient
β	brushes coverage factor
γ	constant of equivalent synchronous generated m.m.f.
δ	air gap; layer thickness
ε	dielectric constant
ζ	Pichelmayer-factor
η	efficiency; dynamic viscosity
θ	electric current linkage
ϑ	load angle; temperature; over temperature
κ	electric conductivity

λ power factor, thermal conductivity; wave length; ordinal number; reduced magnetic conductivity

Λ magnetic conductivity

μ permeability; ordinal number

υ ordinal number; kinematic viscosity

ξ winding factor

ρ specific resistance

σ leakage factor; tensile stress

τ general partition; tangential force

Φ magnetic flux

φ phase displacement between voltage and current

ψ flux linkage

ω angular frequency

6.2 Units

The following table contains most important physical variables and their symbols and units to be used. An overview of possible unit conversions is given in the right column additionally.

physical variable	**Symbol**	**SI-unit**	**abbrev.**	**unit conversion**
length	L	Meter	m	
mass	M	Kilogramm	kg	1 t (ton) = 10^3 kg
time	T	Second	s	1 min = 60 s 1 h (hour) = 3600 s
current intensity	I	Ampere	A	
thermodynamic temperature	T	Kelvin	K	temperature difference $\Delta\vartheta$ in Kelvin
celsiustemperature	ϑ	Degree Centigrade	°C	$\vartheta = T - T_0$
light intensity	I	Candela	cd	
area	A	-	m^2	
volume	V	-	m^3	1 l (Liter) = $10^{-3}m^{-3}$
force	F	Newton	N	1 kp (Kilopond) = 9.81 N 1 N = 1 kg·m/s^2
pressure	P	Pascal	Pa	1 Pa = 1 N / m^2 1 at (techn. atm.) = 1 kp / cm^2 = 0.981 bar, 1 bar = 10^5 Pa 1 kp / m^2 = 1 mm WS
torque	M	-	Nm	1 kpm = 9,81 Nm = 9,81 kg·m^2 / s^2

physical variable	**Symbol**	**SI-unit**	**abbrev.**	**unit conversion**
mass moment of inertia	J	-	Kgm^2	1 kgm^2 = 0.102 $kpms^2$ = 1 Ws^3 impetus moment GD^2 GD^2 = 4 J / kgm^2
frequency	F	Hertz	Hz	1 Hz = 1 s^{-1}
angular frequency	ω	-	Hz	$\omega = 2\pi f$
rotational speed	N		s^{-1}	1 s^{-1} = 60 min^{-1}
speed (transl.)	V	-	m / s	1 m / s = 3,6 km / h
power	P	Watt	W	1 PS = 75 kpm / s = 736 W
energy	W	Joule	J	1 J = 1 Nm = 1 Ws 1 kcal = 427 kpm = 4186,8 Ws 1 Ws = 0,102 kpm
el. voltage	U	Volt	V	
el. field strength	E	-	V / m	
el. resistance	R	Ohm	Ω	
el. conductance	G	Siemens	S	
el. charge	Q	Coulomb	C	1 C = 1 As
capacity	C	Farad	F	1 F = 1 As / V
elektr. constant	ε_0	-	F / m	$\varepsilon = \varepsilon_0\varepsilon_r$ ε_r = relative diel.-constant
inductance	L	Henry	H	1 H = 1 Vs / A = 1 Ωs
magn. flux	*f*	Weber	Wb	1 Wb = 1 Vs 1 M (Maxwell) = 10^{-8} Vs = 1 Gcm

physical variable	Symbol	SI-unit	abbrev.	unit conversion
magn. flux density	B	Tesla	T	1 T = 1 Vs / m^2 = 1 Wb / m^2 1 T = 10^4 G (Gauß) 1 G = 10^{-8} Vs / m^2
magn. field strength	H	-	A / m	1 Oe (Oersted) = 10 / 4π A / cm 1 A / m = 10^{-2} A / cm
magn.-motive force	θ	-	A	
magn. potential	V	-	A	
magn. constant	μ_0	-	-	$\mu_0 = 4\pi 10^{-7}$ H / m μ_0 = 1 G / Oe
permeability	μ	-	-	$\mu = \mu_0\mu_r$ μ_r = relative permeability
angle	α	Radiant	rad	1 rad = 1 m / 1 m $\alpha = l_{curve} / r$

7 Literature reference list

Books and scripts listed as follows may exceed the teaching range significantly. Nevertheless they are recommended best for a detailed and deeper understanding of the content of this lecture.

B. Adkins

The general Theory of electrical Machines, Chapman and Hall, London

Ch.V. Jones

The unified Theory of electrical Machines, Butterworth, London

L.E. Unnewehr, S.A. Nasar

Electromechanics and electrical Machines, John Wiley & Sons

Electro-Craft Corporation

DC Motors, Speed Controls, Servo Systems, Pergamon Press

Ch. Concordia

Synchronous Machines, John Wiley, New York

Bahram Amin

Induction Motors – Analysis and Torque Control

Peter Vas

Vector Control of AC Machines, Oxford Science Publications

T.J.E. Miller

Switched Reluctance Motors and their control, Magna Pysics Publishing